Name: _____   Date: _____

Student Exploration: Arithmetic and Geometric Sequences

Vocabulary: arithmetic sequence, common difference, common ratio, explicit formula, geometric sequence, sequence

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
Ms. Lin wrote this list of numbers on the board: 3, 7, 11, 15, 19. She asked her students to come up with two different ways to tell someone how to write this exact list in order (without just listing the numbers). Fill in the blanks with two different sets of instructions that could work.

1. _____

   _____

2. _____

   _____

Gizmo Warm-up
Ms. Lin's list is an ordered list of numbers, so it is a **sequence**. In fact, it is an **arithmetic sequence** because the difference between any two consecutive numbers (terms) is the same. In the *Arithmetic and Geometric Sequences* Gizmo, you will explore arithmetic and geometric sequences as graphs and lists.
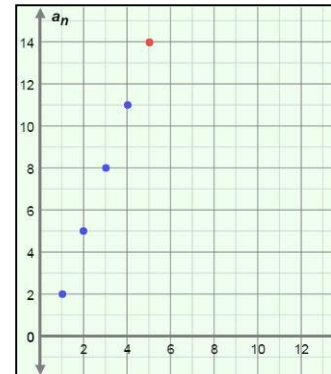
On the **ARITHMETIC** tab, graph Ms. Lin's sequence by setting **First term (a_1)** to 3, **Common difference (d)** to 4, and n (the number of terms) to 5. (To set a value, either drag the slider, or click the text field, type the new value, and hit **Enter**.) To see all the points, either click and drag the graph, or click zoom out (**–**).
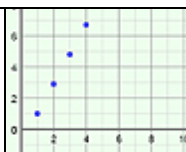

1. Vary d. How does the graph change? _____

   _____

2. Select the **TABLE** tab above the graph.

   A. Vary d. How do the values change? _____

   B. Vary n. What changes? _____

Activity A: Arithmetic sequences	Get the Gizmo ready: • On the **ARITHMETIC** tab, set a_1 to 1, **d** to 2, and **n** to 5.	

1. On the **TABLE** tab, using the values above, look at the a_n (read "*a* sub *n*") column.

   A. What are the five terms of this arithmetic sequence? _____

   B. Select the **GRAPH** tab. Adjust the graph to see all five points. Mouseover the points.

   What are the coordinates? _____

   C. What does each *x*- and *y*-coordinate represent?

   x-coordinate: _____   *y*-coordinate: _____

   The points on the graph of a sequence are called (n, a_n) instead of (x, y).

   D. Find the following differences. (Note: a_1 means "1st term," a_2 "2nd term," and so on.)

   $a_2 - a_1 =$ _____ $a_3 - a_2 =$ _____ $a_4 - a_3 =$ _____ $a_5 - a_4 =$ _____

   You should have found that the differences between consecutive terms are all equal. This value (in this case, 2) is the **common difference** (*d*) of the arithmetic sequence.

   E. Fill in the blanks below to show how many 2's (*d*) you can add to a_1 to get each term.

   $a_2 = a_1 +$ _____ • 2 $a_3 = a_1 +$ _____ • 2 $a_4 = a_1 +$ _____ • 2 $a_5 = a_1 +$ _____ • 2

   F. Use the pattern above to write a general formula for the nth term of an arithmetic sequence (a_n), using the first term (a_1) and the common difference (*d*).

   $a_n =$ _____   This is called an **explicit formula**.

2. An arithmetic sequence has a first term of 2 and a common difference of 0.5. (Do not graph this sequence in the Gizmo yet.)

   A. What are the first six terms of this sequence? _____

   B. If you were to graph these terms, what would the coordinates of the six points be?

   _____   Check this in the Gizmo.

   C. The points of this graph lie on a line. What is the slope of the line? _____

   Explain why that makes sense. _____

(Activity A continued on next page)

Activity A (continued from previous page)

3. On the previous page, you looked at an arithmetic sequence with a first term of 2, and a common difference of 0.5.

   A. What is the explicit formula for this arithmetic sequence? _____

   B. In the space to the right, simplify the formula you wrote above.

   C. How does the simplified formula above relate to the graph of this sequence?

   _____

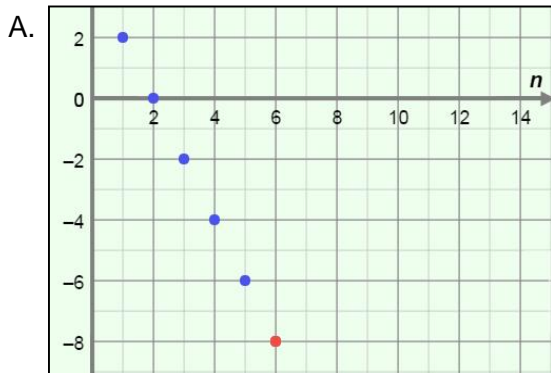4. An arithmetic sequence begins with the terms 5, 2, –1, –4, … .

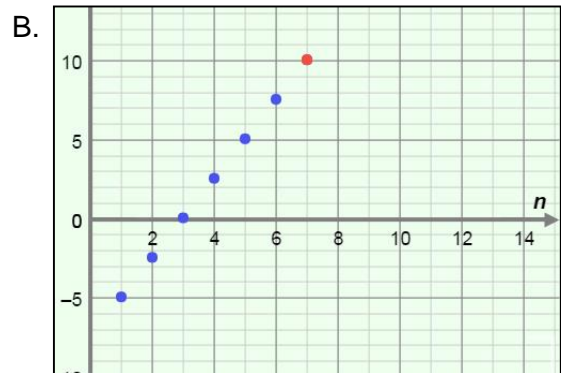   A. What is a_1 of the sequence? _____   What is d? _____

   B. What are the next three terms of this sequence (a_5, a_6, and a_7)? _____

   C. What is the explicit formula for this sequence? _____

   D. What is a_{10}? a_{10} = _____   Check your answer in the Gizmo.

5. Write the explicit formula for the arithmetic sequence shown in each graph. Then check your answers in the Gizmo.

   A.

   _____

   B.

   _____

n	a_n
1	2
2	6
3	18
4	54
5	162

Activity B:	Get the Gizmo ready:
Geometric sequences	• Select the **GEOMETRIC** and **TABLE** tabs. • Set a_1 to 2, r to 3, and n to 5.

1. The values in the a_n column of the table are terms of a **geometric sequence**. In general, the ratio of any two consecutive terms of a geometric sequence is the same.

   A. Using the values above, what are the 5 terms of this sequence? _____

   B. How can you tell that this sequence is not an arithmetic sequence? _____

   _____

   C. Select the **GRAPH** tab. Adjust the graph to see all five points. Mouseover the points.

   What are the coordinates? _____

   D. Find the following ratios. (Note: a_1 means "1st term," a_2 "2nd term," and so on.)

   $a_2 \div a_1 =$ _____ $a_3 \div a_2 =$ _____ $a_4 \div a_3 =$ _____ $a_5 \div a_4 =$ _____

   You should have found that the ratios of consecutive terms are all equal. This value (in this case, 3) is called the **common ratio** (r) of this geometric sequence.

   E. Fill in the blanks with the number you can multiply by the first term, a_1, to get each of the other terms (a_2, a_3, a_4, a_5) in the sequence.

   $a_2 = a_1 \cdot$ _____ $a_3 = a_1 \cdot$ _____ $a_4 = a_1 \cdot$ _____ $a_5 = a_1 \cdot$ _____

   F. Rewrite the equations above using 3 (the value of r) raised to an exponent.

   $a_2 = a_1 \cdot$ _____ $a_3 = a_1 \cdot$ _____ $a_4 = a_1 \cdot$ _____ $a_5 = a_1 \cdot$ _____

   G. Use the pattern above to write a general formula for the nth term of a geometric sequence (a_n), using the first term (a_1) and the common ratio (r).

   $a_n =$ _____   This is the explicit formula for a geometric sequence.

2. A geometric sequence has a first term of 8 and a common ratio of 0.5. (Do not graph this sequence in the Gizmo yet.)

   A. What are the first six terms of this sequence? _____

   B. If you were to graph these terms, what would the coordinates of the six points be?

   _____   Check this in the Gizmo.

(Activity B continued on next page)

Activity B (continued from previous page)

3. On the previous page, you looked at a geometric sequence with $a_1 = 8$ and $r = 0.5$.

    A. What is the explicit formula for this geometric sequence? _____

    B. The points of the graph of this sequence lie on the graph of an exponential function.

       Explain why this makes sense. _____

       _____

    C. Why will the terms in this sequence never reach zero? _____

       _____

4. Set a_1 to 3, r to 1, and n to 10. Vary the r slider for values greater than 1.

    A. Describe the graph from left to right. _____

       _____

    B. Vary the r slider for values between 0 and 1. How does the graph change? _____
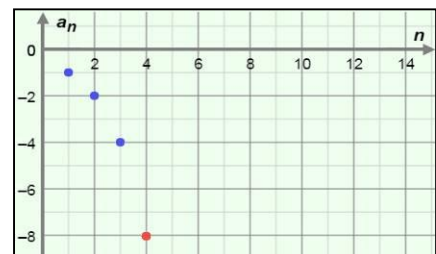
       _____ Why? _____

       _____

    C. Describe the graph when r is negative. _____

       _____ Why is this? _____

       _____

5. Explain why a sequence like 12, 12, 12, 12, … is both arithmetic and geometric.

    _____

    _____

6. Write the explicit formula for the geometric sequence
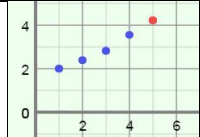   shown in the graph to the right.

   _____

   Then check your answer in the Gizmo.

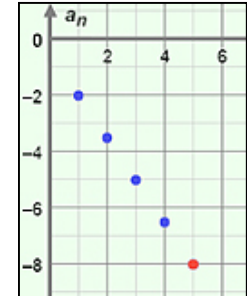| **Activity C:** **Practice with sequences**	<u>Get the Gizmo ready</u>: • Select the **GRAPH** tab.	

1. The graph of a sequence is shown to the right.

   A. Is the sequence arithmetic, geometric, or neither? _____

      Why? _____

      _____

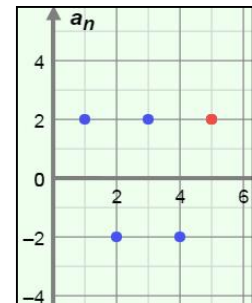   Graph the sequence in the Gizmo to check your answer.

   B. What is the explicit formula for this sequence? _____

2. The graph of a sequence is shown to the right.

   A. Is the sequence arithmetic, geometric, or neither? _____

      Why? _____

      _____

      Check your answer in the Gizmo.

   B. What is the explicit formula for this sequence? _____

3. A sequence starts –5, 1, … . Answer the questions below, and check answers in the Gizmo.

   A. What are the next four terms and explicit formula if this sequence is arithmetic?

      Next 4 terms: _____   Explicit formula: _____

   B. What are the next four terms and explicit formula if this sequence is geometric?

      Next 4 terms: _____   Explicit formula: _____

4. Write an explicit formula for each sequence described below. Then check your answers in the Gizmo. (Recall that "d" means "common difference" and "r" means "common ratio.")

   A. An arithmetic sequence with $d < 0$ and $a_8 = 0$. $a_n =$ _____

   B. A geometric sequence with $r < 0$ and $a_6 > 0$. $a_n =$ _____

   C. A sequence with all positive terms, approaching zero. $a_n =$ _____