



Name: _____

Date: _____

Student Exploration: Binomial Probabilities

Vocabulary: binomial coefficient, binomial experiment, factorial, Pascal's Triangle, tree diagram

Prior Knowledge Question (Do this BEFORE using the Gizmo.)

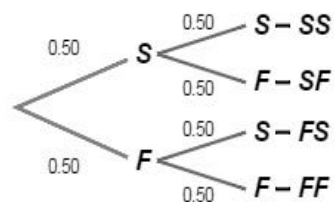
1. Stan is hanging out on the corner, flipping a coin to pass the time. He flips the coin twice, and it comes up "heads" both times. Amazed, Stan runs to tell his friends.

- A. What are the outcomes that can occur if Stan flips the coin once? _____
- B. What outcomes are possible for two flips? _____

2. What do you think is the probability he will get two heads in a row? _____

Gizmo Warm-up

When you flip a coin, there are two possible outcomes: heads or tails. If you flip a coin several times, you are doing a **binomial experiment**. You can explore patterns in the outcomes of binomial experiments with the *Binomial Probabilities* Gizmo.



To begin, check that the **Number of trials** (n) is 2, **$P(\text{Success})$** ($P(S)$) is 0.50, and **$P(\text{Failure})$** ($P(F)$) is 0.50. (To quickly set the value of a slider, type the number into the text box to the right of the slider and press **Enter**.)

1. Notice the **tree diagram** on the right side of the Gizmo. The diagram shows the possible outcomes of two binomial experiments.

- A. What are the two possible outcomes of the first trial? _____
- B. What are the two possible outcomes of the second trial? _____
- C. What are the four possible outcomes of the two trials? _____

2. On the left side of the Gizmo, select the **CALCULATIONS** tab. Select **$P(2 \text{ Successes})$** .

- A. Of the four outcomes, how many have 2 successes? _____
- B. In two trials, what is the probability of 2 successes? _____
- C. Select **$P(1 \text{ Success})$** . What is the probability of 1 success? _____



Activity A: Calculating probabilities	<u>Get the Gizmo ready:</u> <ul style="list-style-type: none"> • On the CONTROLS tab, set n to 2. • Set $P(S)$ to 0.70. 	
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- Imagine a basketball player makes 70% of her free throws ($P(S) = 0.70$).
 - What is the probability she will make her first free throw? _____
 - What do you think is the probability she will make two free throws in a row? _____
 - On the **CALCULATIONS** tab, select **P(2 Successes)** to check your answer. Based on the calculation shown, how do you calculate the probability of two successes?

- In general, the probability of two independent events is the probability of the first event times the probability of the second event. You can see these probabilities (0.70 and 0.30 in the case of the free-throw shooter) along the branches of the tree diagram.
 - What is the probability that the player will make her first shot and miss her second (success, then failure)? $P(SF) =$ _____
 - What is the probability she will miss (F), then make (S)? $P(FS) =$ _____
 - What do you notice about $P(SF)$ and $P(FS)$? _____
 - Add $P(SF)$ and $P(FS)$. What is the probability of one success in two trials? _____

Check your answer by selecting **P(1 Success)** on the **CALCULATIONS** tab.

- Suppose you want to know the probability of making exactly 2 free throws out of 3 free attempts. On the **CONTROLS** tab, set n to 3. (Keep $P(S)$ equal to 0.70.)
 - Look at the tree diagram. Notice there are three ways to get exactly 2 successes: SSF , SFS , and FSS . Find the probability of each of these. Show your work.

$P(SSF) =$ _____ $P(SFS) =$ _____ $P(FSS) =$ _____

- Add your three probabilities to find the probability of 2 successes in three trials.
 $P(2 \text{ Successes}) =$ _____ Check your answer on the **CALCULATIONS** tab.

(Activity A continued on next page)

Activity A (continued from previous page)

4. Suppose you wanted to know the probability of the same player ($P(S) = 0.70$) making exactly 3 free throws in 4 attempts. On the **CONTROLS** tab, set n to 4.

A. What are the different ways that 3 successes out of 4 trials could be arranged?

B. Find the probability of each of these outcomes. Show your work.

C. What do you notice about these probabilities? _____

D. What is $P(3 \text{ Successes})$? _____ Use the **CALCULATIONS** tab to check.

5. In the last question, you may have noticed that the probability of each way of getting 3 successes in 4 trials was the same. You can use this fact to create a general rule.

A. Suppose the probability of success is $P(S)$ and the probability of failure is $P(F)$. What is the probability of 5 successes followed by 2 failures? (Hint: Use exponents.)

$$P(SSSSSFF) = \underline{\hspace{2cm}}$$

B. Suppose you wanted to find the probability of r successes in n trials. First, find the probability of r successes followed by $n - r$ failures. _____

C. Recall that the probability of each way to get r successes in n trials is the same. If there are x different ways you can arrange r successes out of n trials, what is the formula for the probability of r successes in n trials?

$$P(r \text{ Successes}) = \underline{\hspace{2cm}}$$

6. Instead of x , mathematicians use the symbol ${}_n C_r$ to represent the number of ways to arrange r successes in n trials. This term is known as the **binomial coefficient**.

If the probability of success is 0.70 and ${}_5 C_3$ is 10, what is the probability of making exactly 3 free throws in 5 attempts? Show your work below. Then check your answer in the Gizmo.

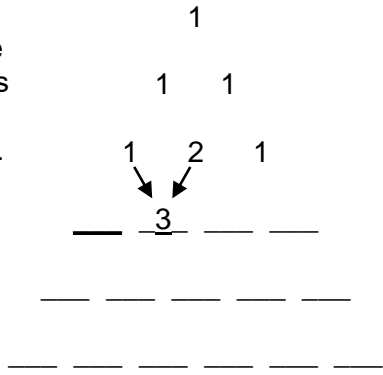
$$P(3 \text{ Successes}) = \underline{\hspace{2cm}}$$



Activity B: The binomial coefficient	<u>Get the Gizmo ready:</u> <ul style="list-style-type: none"> On the CONTROLS pane, set n to 3. Set $P(S)$ to 0.50. 	
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Introduction: The binomial coefficient occurs in many places in probability and statistics. The binomial coefficient $(n C_r)$ gives the number of ways you can arrange r successes in n trials.

1. One way to find the binomial coefficient is to use **Pascal's Triangle**. Pascal's Triangle models how many ways there are to reach various outcomes. To calculate each term in Pascal's Triangle, add the two terms diagonally above it. For example, the "3" on the fourth row is the sum of the "1" and "2" above it.



- A. The first three rows of Pascal's Triangle have been computed. Fill in the next three rows by adding the terms above, as shown.
- B. On the **CALCULATIONS** tab, find the values of ${}_3C_0$, ${}_3C_1$, ${}_3C_2$, and ${}_3C_3$.

List them here. _____

How do these compare to the 4th row of Pascal's Triangle? _____

- C. Set n to 4, and find ${}_4C_0$, ${}_4C_1$, ${}_4C_2$, ${}_4C_3$, and ${}_4C_4$. _____

How do these compare to the 5th row of Pascal's Triangle? _____

- D. Based on Pascal's Triangle, what is ${}_5C_2$? _____ Check your answer in the Gizmo.

2. Pascal's Triangle is a great way to find the binomial coefficient ${}_n C_r$ when n is small, but not very practical when n is large. The general formula for the value of ${}_n C_r$ is given by:

$${}_n C_r = \frac{n!}{r!(n-r)!}$$

The symbol "!" stands for **factorial**, which is the product of a number with all of the positive integers that come before it. For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1$, or 24. By convention, $0! = 1$.

Using the formula, find ${}_5 C_2$. Show your work in the space to the right. When you are done, check your answer in the Gizmo.

${}_5 C_2 =$ _____

Does this result agree with the value obtained from Pascal's Triangle? _____

(Activity B continued on next page)

Activity B (continued from previous page)

3. In activity A, you found that the formula for the probability of r successes in n trials is:

$$P(r \text{ Successes}) = {}_n C_r \cdot P(S)^r \cdot P(F)^{n-r}.$$

What is the general formula for $P(r \text{ Successes})$ in terms of n , r , $P(S)$, and $P(F)$? (Substitute in the general formula for ${}_n C_r$, in terms of n and r .)

$$P(r \text{ Successes}) =$$

4. A professional bowler gets a strike 60% of the time. What is the probability he will get four strikes in a row? Show your work. When you are done, check your answer in the Gizmo.
5. If two parents have 6 children, what is the probability that they will have exactly 5 daughters and 1 son? (Assume that boys and girls are equally likely.) Show your work.
6. A baseball player has a 0.250 batting average. What is the probability he will get exactly four hits in his next ten at-bats? Show your work.
7. A basketball player makes 80% of her free throws. If she takes 8 free throws in a game, what is the probability she will make at least 6 of them? (Hint: Add the probabilities of making 6, 7, and 8 free throws.)

