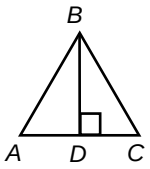
Gizmos

Name: _____ Date: _____

Student Exploration: Concurrent Lines, Medians, and Altitudes

Vocabulary: altitude, bisector, centroid, circumcenter, circumscribed circle, concurrent, incenter, inscribed circle, median (of a triangle), orthocenter

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)



1. A **bisector** is a line, segment, or ray that divides a figure into two congruent parts. The midpoint of a segment lies on its bisector and is halfway between the endpoints.

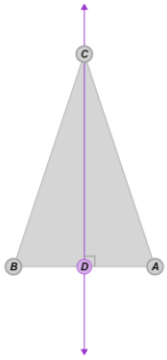
   A. In the triangle, $\overline{BD}$ bisects $\angle ABC$. Which angles are congruent? _____

   B. In this triangle, $\overline{BD}$ also bisects $\overline{AC}$. What is the midpoint of $\overline{AC}$? _____

2. Two intersecting lines are perpendicular if they meet at a right angle. Which two line segments in the figure shown above are perpendicular? _____

Gizmo Warm-up

In the *Concurrent Lines, Medians, and Altitudes* Gizmo, you can manipulate triangles to discover relationships among their bisectors, medians, and altitudes.



1. In the Gizmo, select **Perpendicular bisectors** and **…of $\overline{AB}$**. Drag the vertices (points *A*, *B*, and *C*) to create a variety of triangles.

   A. The line you see in the figure is the perpendicular bisector of $\overline{AB}$.

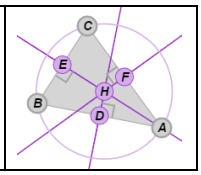
   Does this line always go through point *C*? _____

   B. Select **Show ruler** to open a Gizmo ruler. To measure a segment, attach the ruler's "donuts" to its endpoints. Use the rulers to find the lengths of $\overline{DB}$ and $\overline{DA}$. What do the lengths of these segments tell you about point *D*?

   _____

2. How is a perpendicular bisector related to the segment it intersects? _____

   _____

Activity A: **Perpendicular bisectors and angle bisectors**	Get the Gizmo ready: • Be sure **Perpendicular bisectors** is selected. • Turn on all three perpendicular bisectors.	

1. The perpendicular bisectors are the three lines that are perpendicular to the sides of the triangle and go through the midpoint of each side.

   A. Do all of the perpendicular bisectors meet at a point? _____

   Create a variety of triangles to check if this is always true.

   B. When several lines meet at a point, they are **concurrent**. The point where the three perpendicular bisectors meet is called the **circumcenter** of the triangle.

   In the Gizmo, what point is the circumcenter? _____

   C. In the Gizmo, create a variety of triangles. Watch how the location of the circumcenter changes. Write *interior*, *exterior*, or *on* in the last column of the table to tell where the circumcenter lies in relationship to each type of triangle listed in the first column.

Type of triangle	Location of circumcenter
Acute	
Right	
Obtuse	

2. With **Perpendicular bisectors** selected and all three perpendicular bisectors turned on, select **circumscribed circle**. A **circumscribed circle** is a circle on which all of the vertices of a triangle lie.

   A. Compare the words “circumscribed” and “circumcenter.” Why do you think the point of intersection of the perpendicular bisectors of a triangle is called the circumcenter?

   _____

   _____

   B. How do the distances from each vertex to the circumcenter compare? Why?

   _____

   _____

   _____

   Use the Gizmo rulers to check. Then view more triangles to see if this is always true.

(Activity A continued on next page)

Activity A (continued from previous page)

3. Turn off **Perpendicular bisectors** and select **Angle bisectors**. Turn on the angle bisectors of angles *A*, *B*, and *C*.

   A. Are the angle bisectors concurrent? _____ In the Gizmo, create a variety of triangles to check if this is always true.

   B. What is the point of concurrency of the angle bisectors in each triangle? _____ This point is called the **incenter** of a triangle.

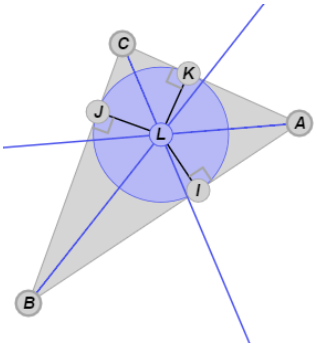
   C. Create a variety of triangles. What do you notice about the location of the incenter?

   _____

   D. The prefix of the term "incenter" is "in." Why do you think this term accurately describes the location of the incenter of a triangle? _____

   _____

4. With **Angle bisectors** selected and all three angle bisectors turned on, select **inscribed circle**. An **inscribed circle** fits inside a triangle and touches each side at exactly one point.



   A. What part of the inscribed circle is the incenter, point *L*?

   _____

   In the Gizmo, create and test other triangles to see if this is always true.

   B. What parts of the inscribed circle are $\overline{LI}$, $\overline{LK}$, and $\overline{LJ}$? _____

   _____

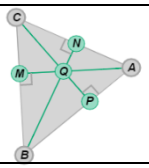
   C. What is true about the lengths of $\overline{LI}$, $\overline{LK}$, and $\overline{LJ}$? _____ Check with the Gizmo rulers. Then test other triangles to see if this is always true.

   D. How are $\overline{LI}$, $\overline{LK}$, and $\overline{LJ}$ each related to the side of the triangle they intersect?

   _____

   E. What can you say about the distance from the incenter to each side of the triangle?

   _____

Activity B: **Altitudes and medians**	Get the Gizmo ready: • Turn off **Angle bisectors**. • Select **Altitudes**. • Turn on all three altitudes.	

1. An **altitude** is a line that passes through a vertex of a figure and is perpendicular to the opposite side.

   A. Are the altitudes concurrent? _____ In the Gizmo, check a variety of triangles.

   B. What is the point of concurrency of the altitudes of a triangle? _____

      This point is called the **orthocenter** of a triangle.

   C. In the Gizmo, create a variety of triangles. Watch how the location of the orthocenter changes. Write *interior*, *exterior*, or *on* in the last column of the table to tell where the orthocenter lies in relationship to each type of triangle listed in the first column.

Type of triangle	Location of orthocenter
Acute	
Right	
Obtuse	

2. Be sure **Altitudes** is selected and all three altitudes are turned on.

   A. Drag the vertices to create a right triangle with right $\angle A$. Make a labeled sketch of the right triangle in the space to the right.

   B. What happens to points *P*, *N*, and *Q* when a right triangle is formed? _____

      _____

   C. Look at the altitudes and the legs of the right triangle. What do you notice? _____

      _____

      In the Gizmo, create a variety of right triangles to check if this is always true.

   D. Think about the definition of altitude. Why do all three altitudes all meet at the right angle of the triangle? _____

      _____

      _____

(Activity B continued on next page)

Activity B (continued from previous page)

3. Turn off **Altitudes** and select **Medians**. Turn on all three medians. A **median** of a triangle is a line that passes through a vertex and the midpoint of the opposite side. In the Gizmo, create a variety of triangles and watch what happens to the medians.

   A. Are the medians always concurrent? _____

   B. What is the point of concurrency of the medians for each triangle? _____

   This point is called the **centroid** of a triangle.

   C. Is the centroid sometimes, always, or never in the interior of a triangle? _____

4. With all three medians still showing, turn on both Gizmo rulers.

   A. Create any triangle in the Gizmo. Use the rulers to measure the segments listed in the table below. Write the measures in the second row.

Segment	***AZ***	***AR***	***BZ***	***BS***	***CZ***	***CT***
Measure						

   B. Use a calculator to write each of the following ratios as a decimal, and then as a simplified fraction of integers.

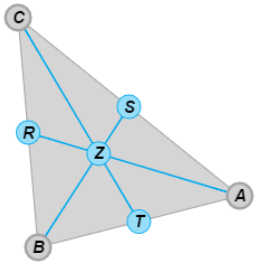
   $\frac{AZ}{AR}$ = _____ $\frac{BZ}{BS}$ = _____ $\frac{CZ}{CT}$ = _____

   C. What do you notice? _____

   D. In the Gizmo, create another triangle and check these ratios. Complete the ratio to the right for any triangle.

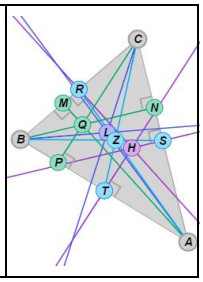
$$\frac{\text{distance from vertex to centroid}}{\text{length of median from that vertex}} = \frac{\square}{\square}$$

5. The medians of ΔABC to the right are $\overline{AR}$, $\overline{BS}$, and $\overline{CT}$. Use the ratio from above to answer the following questions. Show all of your work.



   A. If $AR = 12$, what is AZ?

   B. If $CZ = 9$, what is CT?

Extension: **The Euler line**	Get the Gizmo ready: • Drag the vertices to create a scalene triangle. • Turn on **Perpendicular bisectors**, **Angle bisectors**, **Altitudes**, and **Medians**. • Turn on every perpendicular bisector, angle bisector, altitude, and median.	

1. Focus on the circumcenter (point *H*), the incenter (point *L*), the orthocenter (point *Q*), and the centroid (point *Z*).

   A. What do you notice about these points? _____

   _____

   B. Move the triangle vertices to create a variety of triangles. What do you notice about points *H*, *L*, *Q*, and *Z* as you do this? _____

   _____

2. Move the vertices of the triangle to create an obtuse scalene triangle. Points *H* and *Q* should be outside the triangle. Turn on **Show ruler** and connect points *H* and *Q*.

   A. What other point or points lie on the line between *H* and *Q*? _____

   B. Move the triangle vertices to create a variety of triangles. Is this always true? _____

   Explain. _____

3. The segment between the circumcenter and the orthocenter lies on a line called the *Euler line*. The Euler line connects some of the important points of a triangle.

   A. What three points are always on the Euler line? (Use the full names for these points.)

   _____

   B. What point does not always lie on the Euler line? _____

   C. Manipulate the triangle until this point lies on the Euler line. For what type of triangles does this point lie on the Euler line? _____

   D. Drag the vertices to create an equilateral triangle. Why is the Euler line undefined for equilateral triangles? _____

   _____