Gizmos

Name: ______________________________ Date: ____________________

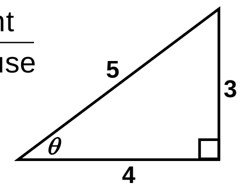
Student Exploration: Cosine Function

Vocabulary: cosine, even function, period, radian, reference triangle, trigonometric function, unit circle

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

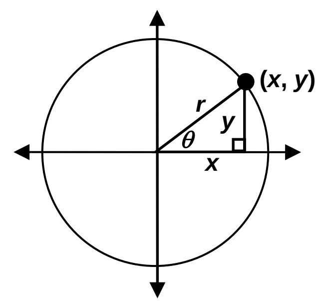
1. The **cosine** of angle θ of a right triangle is the length of the adjacent leg divided by the hypotenuse.

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$$



The hypotenuse of the right triangle to the right is 5 units, and the legs have lengths of 3 units and 4 units.

What is the cosine of angle θ? $\cos(\theta)$ = __________



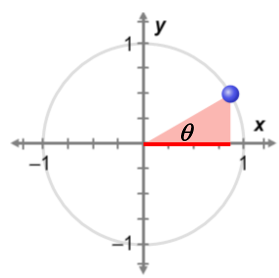
2. A circle has its center at the origin of a coordinate plane. A right triangle is placed in the circle as shown to the right.

   A. What is $\cos(\theta)$? $\cos(\theta)$ = __________

   B. What is $\cos(\theta)$ if $r = 1$? $\cos(\theta)$ = __________

Gizmo Warm-up

The cosine function $y = \cos(\theta)$ is a **trigonometric function**. When θ has its vertex at the center of a circle, it is in standard position and $\cos(\theta)$ is the x-value of the point where θ intersects the circle. In the *Cosine Function* Gizmo, you will explore $y = \cos(\theta)$ and its graph.



1. On the **COSINE** tab, turn on **Show reference triangle**. Then, with **Degrees** selected, drag the slider slowly from 0° to 180°.

   A. What happens to the value of $\cos(\theta)$ as θ goes from 0° to 180°?

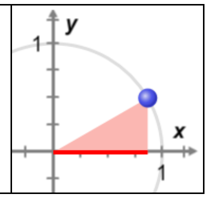
   ____

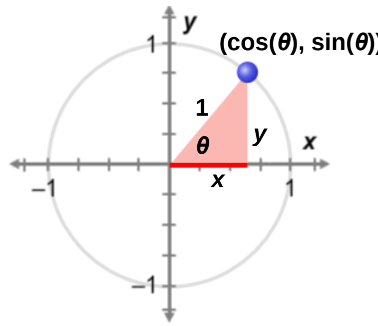
   B. When does the maximum value of $\cos(\theta)$ occur? ____________________

2. Explain why the behavior of $\cos(\theta)$ from 0° to 180° makes sense, based on the unit circle.

____

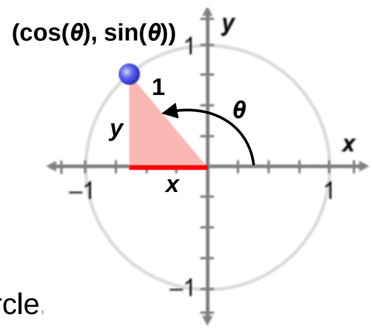
____

Activity A: **The basics of cosine**	<u>Get the Gizmo ready</u>: • On the **COSINE** tab, be sure **Degrees** and **Show reference triangle** are selected. • Set $\boldsymbol{\theta}$ to 0°.	



The circle shown in the Gizmo has a radius of 1, so it is a **unit circle**. Angle θ is formed by a radius of the circle and the positive x-axis. The cosine of θ is the x-value of the corresponding point on the unit circle.

The right triangle formed by the perpendicular segment drawn from the terminal ray of θ to the x-axis is called a **reference triangle**.



1. Set $\boldsymbol{\theta}$ to 0°, so the blue point is at (1, 0). (To quickly set $\boldsymbol{\theta}$ to a specific value, type the value in the text box, and hit **Enter**.) Then drag the point counterclockwise around the circle once.

   A. When is cos(θ) positive? ____________________

   B. When is cos(θ) negative? ____________________

   C. Explain why that makes sense, based on the unit circle.

   ____

   ____

   D. Describe how the x-coordinate changes in one full rotation around the circle.

   ____

   E. What do you think will happen to the value of cos(θ) if you keep dragging the point around the circle? ____

   Why? ____

   ____

   Check your answer in the Gizmo.

   F. How often do the values of the cosine function repeat? ________________________

   This is called the **period** of the cosine function. A function that repeats in regular intervals like this is *periodic*.

(Activity A continued on next page)

Activity A (continued from previous page)

2. Set θ to 180°. Notice that cos(180°) = –1.

   A. List three angles greater than 180° with a cosine of –1. ______________________

   B. List three angles less than 180° with a cosine of –1. ________________________

   C. Justify your answers above. ____

   ____

   D. Drag the point on the unit circle to check your answers above. Then fill in the blanks.

   cos(180°) = cos(180° + _______) = cos(180° + _______) = cos(180° + _______)

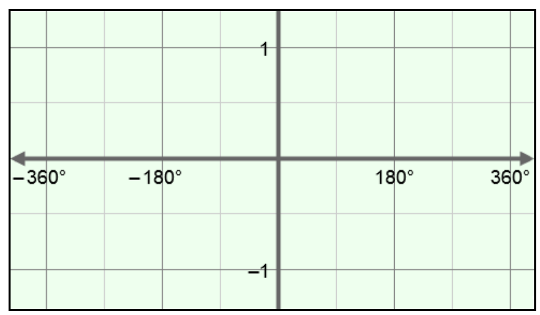
   cos(180°) = cos(180° – _______) = cos(180° – _______) = cos(180° – _______)

   E. In the Gizmo, check that this relationship is true for angles other than 180°. Then fill in the blank to generalize this relationship. cos(θ) = cos($\theta \pm$ (_____n)°)

   F. The cosine function is y = cos(θ). This means that, when you graph it, θ goes on the x-axis and cos(θ) on the y-axis.

   What do you think the graph of y = cos(θ) looks like? Sketch your graph to the right.

   After you are done, select **Show curve** in the Gizmo. Adjust your sketch as needed.



3. Angles can be measured in **radians** instead of degrees. A radian is a unit of angle measure, such that one full rotation (360°) equals 2π radians.

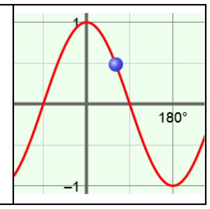
   A. If 360° = 2π, what is the radian measure of a 180° angle? __________

   B. A 60° angle is $\frac{1}{3}$ of 180°. What does 60° equal in radians? ________

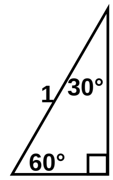
   C. Fill in the radian measure equal to each degree measure below. Check your answers in the Gizmo. (Select **Degrees**, type the degree measure, and select **Radians**.)

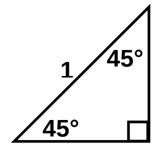
Degree measure	0°	30°	45°	60°	90°
Radian measure					

   D. State the identity cos(θ) = cos($\theta \pm$ (360n)°) using radians. ______________________

Activity B: **The cosine function and identities**	Get the Gizmo ready: • On the **COSINE** tab, be sure **Show curve** and **Show reference triangle** are turned on. • Select **Degrees** and **Common angles only**.	

1. Label the legs of the 30-60-90 and 45-45-90 triangles to the right with their lengths. (Hint: If you don't remember these values, use the Pythagorean Theorem. The short leg of the 30-60-90 triangle is exactly half of the hypotenuse.)





2. Start with θ at 0°, and drag the point on the circle counterclockwise from 0° to 180°.

   A. Fill in the table to the right with the cosine values of 30°, 45°, and 60°.

θ	30°	45°	60°
cos(θ)			

   B. What reference triangle (30-60-90 or 45-45-90) would you use for each angle below?

   30° ________________ 45° ________________ 60° ________________

   C. Fill in the table to the right with the cosine values of 120°, 135°, and 150°.

θ	120°	135°	150°
cos(θ)			

   D. What reference triangle (30-60-90 or 45-45-90) would you use for each angle below?

   120° ________________ 135° ________________ 150° ________________

   E. For the angles above, what is true about cos(θ) for the same reference triangle?

   ____

3. Turn off **Common angles only**. Set θ to 0°. Drag the point around the circle.

   A. In which quadrants is cosine positive? ______________ negative? ______________

   B. Explain why, using the unit circle. ____

   ____

   C. Use what you know about reference triangles and quadrants to find the values.

   cos(225°) = __________ cos(330°) = __________ cos(480°) = __________

(Activity B continued on next page)

Activity B (continued from previous page)

4. Set ***θ*** back to 0°. Drag the point around the circle. Examine pairs of angles whose measures add to 180°, or π radians (for example, 60° and 120°, or 210° and –30°).

   A. What do you notice about their cosine values? ________________________________

   ____

   B. Two angles have a sum of 180°. If one angle is θ, what expression represents the other angle? _________________

   C. Write two equations to show how the cosine values of angles that add to 180° relate to each other. (Write one equation in degrees, and the other in radians.)

   ____

5. Set ***θ*** back to 0°. Drag the point around the circle.

   A. Examine pairs of opposite angles (for example, 30° and –30°). What is true about their cosine values? ____

   B. Use what you observed above to write an equation about the cosine values of opposite angles. ________________________________

   This makes cosine an **even function**, and its graph is symmetric about the *y*-axis.

   C. Examine pairs of angles that are 180° apart (for example, 30° and 210°). What is true about their cosine values? ____

   ____

   D. Use what you noticed to write two equations to show how the cosine values of angles that are 180° apart are related. (Write one in degrees, and one in radians.)

   ____

   E. It is also true that $\cos(\theta) = \cos(360° - \theta) = \cos(2\pi - \theta)$. Explain why this makes sense, using the unit circle.

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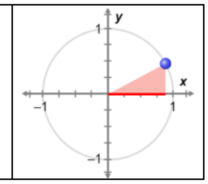
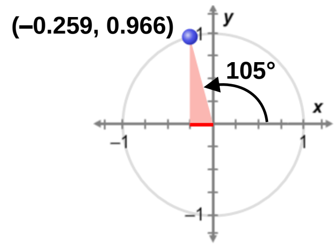
Activity C: **Practice with the cosine function**	Get the Gizmo ready: • On the **COSINE** tab, select **Degrees**.	

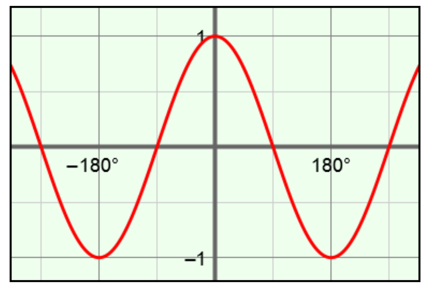
1. The angle shown on the unit circle to the right has a measure of 105°.



   A. What is cos(105°)? ______________________

   B. The graph of $y = \cos(\theta)$ is shown to the right. Plot and label the point that shows cos(105°) on this graph. Check in the Gizmo.

   C. Plot three other points on the graph with a *y*-value (cosine) of –0.259. Write the coordinates of the points below. Check your points in the Gizmo.



   ____

   ____

   D. Plot four points on the graph with a *y*-value of 0.259. Write the coordinates of the points below. Check your points in the Gizmo.

   ____

2. Give the cosine value of each angle below. Then list four different angles (two positive and two negative) with the same cosine value. Check your answers in the Gizmo.

   A. $\cos(\frac{\pi}{6}) =$ ______ Angles with same cosine value: ___________________________

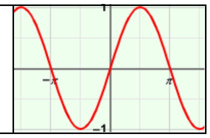
   B. $\cos(\frac{\pi}{4}) =$ ______ Angles with same cosine value: ___________________________

   C. $\cos(\frac{\pi}{3}) =$ ______ Angles with same cosine value: ___________________________

3. Use the fact that cos(50°) ≈ 0.643 to find each value. Check your answers in the Gizmo.

   A. cos(–50°) ≈ ________ C. cos(130°) ≈ ________ E. cos(310°) ≈ ________

   B. cos(–410°) ≈ _______ D. cos(–230°) ≈ _______ F. cos(410°) ≈ ________

Extension: **Cosine and sine**	Get the Gizmo ready: • Select the **SINE** tab.	

1. The sine of angle θ in a right triangle is the length of the opposite leg divided by the hypotenuse. On the unit circle, $\sin(\theta)$ is the y-value of the point where θ intersects the circle.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

Drag the point counterclockwise. How does the y-coordinate change in one full rotation?

____

____

2. Set ***θ*** to 0°. Drag the point around the circle. Examine pairs of angles whose measures add to 90°. (Be sure to look at angles with both positive and negative measures.)

   A. What do you notice about the cosine of one angle and the sine of the other?

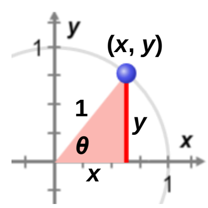
   ____

   B. Two angles add to 90°. If one angle is θ, what is the other angle? _______________

   C. Write two equations to summarize how the cosine and sine values of angles that add to 90° relate to each other.

   $\cos(\theta)$ = _______________________ $\sin(\theta)$ = __________________________

3. If a and b are the legs of a right triangle with hypotenuse c, then the Pythagorean Theorem states that $a^2 + b^2 = c^2$.



   A. Use the Pythagorean Theorem to write an equation for the reference triangle shown to the right. _______________________

   B. Use $\cos(\theta)$ and $\sin(\theta)$ to write the *Pythagorean Identity*. _______________________

4. Use the Pythagorean Identity to find each value. Show your work, and check in the Gizmo.

   A. $\cos(\theta) = \frac{\sqrt{3}}{2}$ $\sin(\theta)$ = ____________ B. $\sin(\theta) = 0.391$ $\cos(\theta) \approx$ ___________