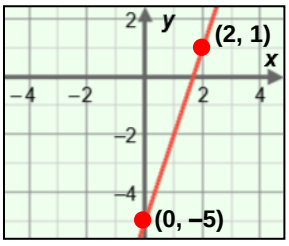
Name: _____ Date: _____

Student Exploration: Graphs of Derivative Functions

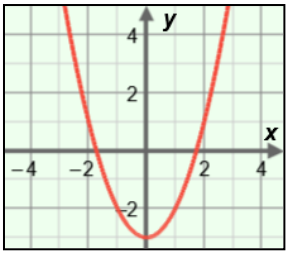
Vocabulary: derivative



Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. The slope of a line tells you the rate of change of *y*, relative to *x*.

   What is the slope of the line shown here? _____



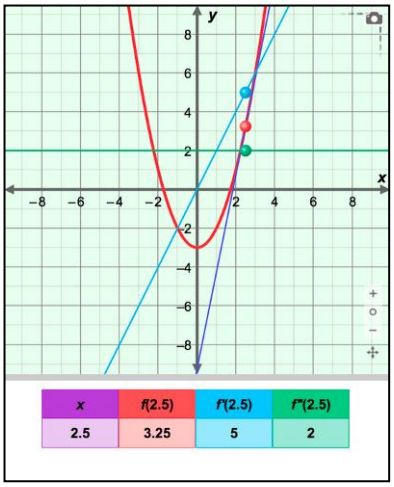
2. The graph of $y = x^2 - 3$ is shown to the right. In general, how is the "slope" of a parabola different from the slope of a line?

   _____

   _____

Gizmo Warm-up

The **derivative** of a function *f*(*x*), denoted *f'*(*x*), is the rate of change of the function at a point. If the graph is a curve, the derivative is the slope of the tangent line. In the *Graphs of Derivative Functions* Gizmo, you will find the derivatives of several functions, and explore the graphs of derivative functions.



At the top left of the Gizmo, select **Linear function**. Set ***a*** to 3 and ***b*** to –5 to graph $y = 3x - 5$. (To quickly set a specific value, type the value in the text box, and hit **Enter**.)

1. What do you think the derivative of $f(x) = 3x - 5$ is? _____

   Why? _____

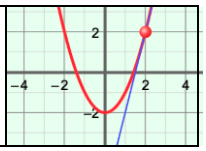
   _____

   Select **Show derivative** to check your answer.

2. Vary ***b***. Explain why changing the value of *b* does not affect the derivative. _____

   _____

   _____

Activity A: **Quadratics and cubics**	Get the Gizmo ready: • Select **Quadratic function** and **Show function**. • Turn off **Show derivative**.	

1. Set ***a*** to 1, ***b*** to 0, and ***c*** to –2 to graph $f(x) = x^2 - 2$. Take a look at its graph in the Gizmo.

   A. Where is the "slope" of the graph of $f(x) = x^2 - 2$:

      positive? _____

      negative? _____

   B. The "slope" of a curve at a point is the slope of the line tangent to the curve at that point. (This is usually called the derivative.) Select **Show tangent line**. Drag the red point along the parabola, and watch the blue tangent line as you do.

      Where is the slope of the tangent line zero? _____

      Explain why this makes sense. _____

      _____

2. Graph $f(x) = 0.5x^2 - 4$. Select **Show derivative**. Drag the red point. The *y*-values on the light blue line give you the slope of the dark blue tangent line at the current *x*-value.

   A. What does the light blue line tell you about the derivative of $f(x) = 0.5x^2 - 4$?

      _____

      _____

   B. Vary ***a***, ***b***, and ***c***. What type of function is the derivative of a quadratic? _____

   C. Vary ***c***. How does *c* affect the derivative? _____

      Explain why this makes sense. _____

      _____

   D. Set ***b*** and ***c*** to 0. Vary ***a***. In general, what is the derivative of $f(x) = ax^2$? _____

      This is an example of the *power rule*: the derivative of $f(x) = x^n$ is $f'(x) = nx^{(n-1)}$.

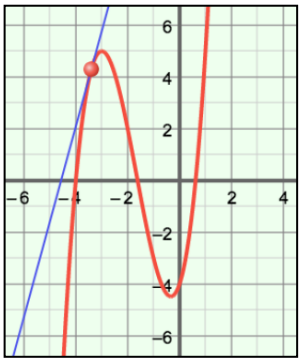
   E. Vary ***a***, ***b***, and ***c***. Look for a pattern in how these values affect the derivative.

      In general, what is the derivative of $f(x) = ax^2 + bx + c$? $f'(x) =$ _____

(Activity A continued on next page)

Activity A (continued from previous page)

3. With **Show tangent line** still selected, turn off **Show derivative**. Select **Cubic function**, and graph $f(x) = x^3 + 5x^2 + 3x - 4$.



   A. The graph should look like the one shown. Where is the "slope" of $f(x) = x^3 + 5x^2 + 3x - 4$:

      positive? _____

      negative? _____

   B. Drag the red point along the curve, and watch how the dark blue tangent line changes.

      Where is the slope of the tangent line zero? _____

      Explain why this makes sense. _____

      _____

4. Graph $f(x) = 0.4x^3 + 2$. Select **Show derivative**, and drag the red point along the curve.

   A. What does the light blue curve tell you about the derivative of $f(x) = 0.4x^3 + 2$?

      _____

      _____

   B. Vary ***a***, ***b***, ***c***, and ***d***. What type of function is the derivative of a cubic? _____

   C. Vary ***d***. How does *d* affect the derivative? _____

      Explain why. _____

   D. Vary ***a***, ***b***, ***c***, and ***d***, and look for a pattern in the derivative. In general, what is the derivative of $f(x) = ax^3 + bx^2 + cx + d$? $f'(x) =$ _____

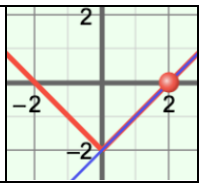
5. Find the derivative of each function. Check your answers in the Gizmo.

   A. If $f(x) = 3x^2 - 4x - 5$, then $f'(x) =$ _____

   B. If $f(x) = -0.3x^2 + 0.5x + 4$, then $f'(x) =$ _____

   C. If $f(x) = -5x^3 + 2x^2 - 3x + 1$, then $f'(x) =$ _____

   D. If $f(x) = 0.4x^3 - 1.5x^2 + 2x - 4$, then $f'(x) =$ _____

Activity B: **Absolute value functions**	Get the Gizmo ready: • Turn off **Show derivative**. • Select **Absolute value function**. • Turn on **Show tangent line**.	

1. Set ***a*** to 1 and ***b*** to –2 to graph $f(x) = |x| - 2$. (Notice that, for absolute value functions, the tangent line is an extension of one part of the graph.) Drag the red point along the graph.

   A. What is the equation of the left half of the graph (where $x < 0$)? _____

   B. What is the equation of the right half of the graph (where $x > 0$)? _____

   C. What is the derivative (slope) of the left half? _____ Of the right half? _____

   D. If the graph of a function has a break in it (a hole or discontinuity), or if it has a sharp turn (like a corner), then the derivative ($f'(x)$) is not defined at that point.

      Where do you think $f'(x)$ for an absolute value function is undefined? _____

   E. Based on what you have seen, how would you write the derivative of $f(x) = |x| - 2$?

      $f'(x)$ = _____ Explain. _____

      _____

      Select **Show derivative** to check. (The light blue graph shows $f'(x)$ at all x-values.)

   F. Vary ***b***. How does b affect the derivative? _____

      Explain why this makes sense. _____

      _____

   G. Vary ***a*** and ***b*** to see other absolute value functions. In general, what is the derivative of $f(x) = a|x| + b$? $f'(x)$ = _____

2. Find the derivative of each function. For A-D, check your answers in the Gizmo.

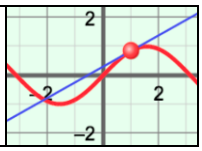
   A. If $f(x) = |x| + 4$, then $f'(x)$ = _____

   B. If $f(x) = -2|x| - 5$, then $f'(x)$ = _____

   C. If $f(x) = 0.5|x| + 3$, then $f'(x)$ = _____

   D. If $f(x) = -1.4|x| + 3.7$, then $f'(x)$ = _____

   E. If $f(x) = 4|x + 3| - 2$, then $f'(x)$ = _____

Activity C: **Sine functions**	Get the Gizmo ready: • Turn off **Show derivative**. • Select **Sine function**.	

1. Set ***a*** to 1, ***b*** to 1, and ***c*** to 0 to graph $f(x) = \sin(x)$. Drag the red point along the sine curve.

   A. Describe the graph of $f(x) = \sin(x)$. _____

   B. Select **Show tangent line**. The blue line is tangent to the curve. Drag the red point again. How does the slope of the tangent line (the derivative, or $f'(x)$) change?

   _____

   _____

   C. Turn on **Show derivative**. The light blue curve shows the values of the derivative at all x-values. What is the derivative of $f(x) = \sin(x)$? $f'(x) =$ _____

2. Turn off **Show tangent line** and **Show derivative**. Then vary the values of ***a*** and ***b***.

   A. How do a and b affect the graph? _____

   _____

   B. Turn on **Show derivative**. Vary ***a*** and ***b*** again. How do the values of a and b affect the derivative of sine functions? (Hint: Vary ***b*** first, with ***a*** set to 1.)

   _____

   _____

   C. Vary ***c***. How does c affect the derivative? _____

   Explain why. _____

   D. Use the sliders one more time to review what you've seen. In general, what is the derivative of $f(x) = a\sin(bx) + c$? $f'(x) =$ _____

3. Find the derivative $f'(x)$ of each function $f(x)$. Then check your answers in the Gizmo.

   A. If $f(x) = 3\sin(x)$, then $f'(x) =$ _____

   B. If $f(x) = \sin(4x) + 5$, then $f'(x) =$ _____

   C. If $f(x) = 2\sin(0.5x) - 3$, then $f'(x) =$ _____

Extension: **The second derivative**	Get the Gizmo ready: • Select **Show second derivative**. • Check that **Show derivative** is selected.	

The second derivative of $f(x)$ is the derivative of the first derivative $f'(x)$. So, in other words, the second derivative $f''(x)$ is the rate at which $f'(x)$ is changing. When the 2nd derivative is negative, the graph of $y = f(x)$ is "concave down," and when it is positive, the graph is "concave up."

1. Be sure you have 3 graphs showing in the Gizmo now. Red is the given function ($y = f(x)$), light blue is the first derivative ($y = f'(x)$), and green is the second derivative ($y = f''(x)$).

   A. Select **Linear function**. Vary ***a*** and ***b***. In general, what is the 2nd derivative of a linear function of the form $f(x) = ax + b$? $f''(x) =$ _____ Why? _____

   _____

   B. Select **Absolute value function**. Vary ***a*** and ***b***. In general, what is the 2nd derivative of $f(x) = a|x| + b$? $f''(x) =$ _____ Why? _____

   _____

   C. Select **Quadratic function**. Vary ***a***, ***b***, and ***c***. In general, what is the 2nd derivative of $f(x) = ax^2 + bx + c$? $f''(x) =$ _____ Why? _____

   _____

   D. Select **Cubic function**. Vary ***a***, ***b***, ***c***, and ***d***. In general, what is the 2nd derivative of $f(x) = ax^3 + bx^2 + cx + d$? $f''(x) =$ _____ Why? _____

   _____

   E. Select **Sine function**. What is the 2nd derivative of $f(x) = \sin(x)$? $f''(x) =$ _____

   What is the 2nd derivative of $f(x) = a\sin(bx) + c$? $f''(x) =$ _____

2. Find the first and second derivatives of each function. Check your answers in the Gizmo.

   A. $f(x) = -4x + 5$ $f'(x) =$ _____ $f''(x) =$ _____

   B. $f(x) = 2x^2 - 3x + 4$ $f'(x) =$ _____ $f''(x) =$ _____

   C. $f(x) = 5x^3 + 0.5x^2 + x - 3$ $f'(x) =$ _____ $f''(x) =$ _____

   D. $f(x) = -3\sin(2x) - 4$ $f'(x) =$ _____ $f''(x) =$ _____