



Name: _____

Date: _____

Student Exploration: Dilations

Vocabulary: dilation, image, matrix, preimage, scalar, scale factor, transformation

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

Marie has a small photograph of a rose. She goes to a photo store and asks them to enlarge the photo to three times its original size.



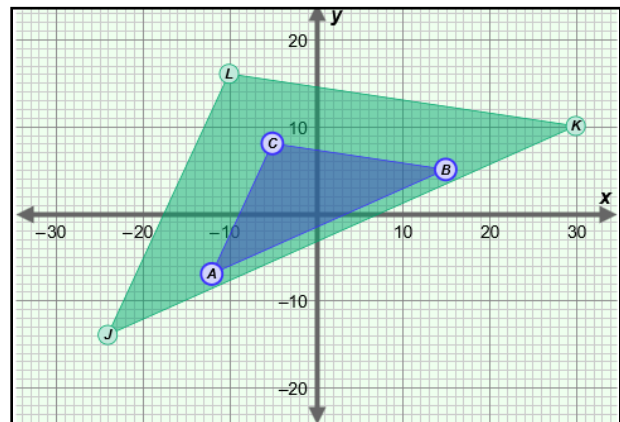
1. How will the enlarged image of the flower be different from the original?

2. How will the enlarged image be the same as the original? _____

Gizmo Warm-up

When you enlarge or reduce the size of a picture, you perform a geometric **transformation** called a **dilation**. In the *Dilations* Gizmo, you can experiment with these transformations by dilating different figures.

Be sure **Point** is selected from the **Figure type** dropdown menu. Drag point A to (8, 6). Look at the **matrix** (rectangular array of numbers) under **Preimage matrix**.



1. How is the preimage matrix related to the coordinates of point A? _____

2. Where will point A be if the preimage matrix is $\begin{bmatrix} -2 \\ 7 \end{bmatrix}$? (_____, _____)

Check your answer in the Gizmo by entering the numbers under **Preimage matrix**. (Click on a number in the text field, type in a new value, and hit **Enter**.)



Activity A: Dilating geometric figures	<u>Get the Gizmo ready:</u> <ul style="list-style-type: none"> • Select Segment from the Figure type menu. • Drag point A to $(-7, 5)$ and point B to $(10, 9)$. 	
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1. With A at $(-7, 5)$ and B at $(10, 9)$, look at the **Preimage matrix**. How is it related to \overline{AB} ?

2. Drag the **Scale factor** slider. \overline{JK} is the dilated **image** of \overline{AB} (the **preimage**). The ratio of the length of \overline{JK} to the length of \overline{AB} is the **scale factor**.

A. What happens to \overline{JK} when the scale factor is greater than 1? _____

B. What happens to \overline{JK} when the scale factor is less than 1? _____

Experiment with a variety of segments to see if this is always true.

3. Drag point A to $(-4, 5)$ and point B to $(9, -8)$. Set **Scale factor** to 2.0. Mouseover points J and K to see their coordinates.

A. How can you use the scale factor of 2 and the endpoints of \overline{AB} to find the endpoints of \overline{JK} ? _____

B. Experiment with a variety of scale factors and watch how the endpoints of \overline{JK} change. If the coordinates of the preimage are (x, y) , write a rule for finding the coordinates of the image of a dilation by a scale factor of r .

$$(x, y) \rightarrow (\underline{\hspace{2cm}}, \underline{\hspace{2cm}})$$

4. Set **Scale factor** to 2.0.

A. How do you think JK compares to AB ? _____

Use the Gizmo rulers to check your answer. (Select **Show ruler**. Attach one set of "donuts" to points A and B , and the other set to points J and K .)

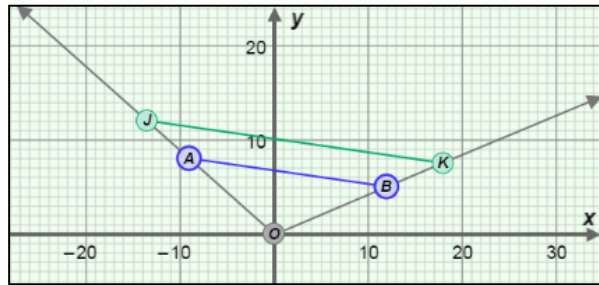
B. Repeat with several other segments and different scale factors. In general, how can you use the scale factor to find the length of the image?

(Activity A continued on next page)



Activity A (continued from previous page)

5. Turn off the Gizmo rulers. Turn on **Show rays**. Drag the endpoints of \overline{AB} until the graph looks like the one to the right.



- A. Where is the endpoint of both rays?

- B. What other points do the rays include? _____

- C. Vary the scale factor. Describe the motion of points J and K . _____

6. Turn off **Show rays**.

- A. What do you think the **Preimage matrix** will look like for a triangle? _____

_____ Set **Figure type** to **Triangle** to check.

- B. Drag the **Scale factor** slider. Describe how the scale factor changes the relationship between the image, $\triangle JKL$, and the preimage, $\triangle ABC$. (Hint: Look at both the sizes of the triangles and the coordinates of their vertices.)

- C. Set **Scale factor** to 1.5. Use the Gizmo rulers to measure corresponding sides of \overline{AB} and \overline{JK} . How does JK relate to AB ? _____

- D. Measure the other pairs of corresponding sides. What do you notice? _____

Experiment with other triangles and scale factors to check that this is always true.

- E. Use the protractors to measure each pair of corresponding angles. (Select **Show angle measure tool** and attach the “donuts” of each protractor.) What do you find?

Create more triangles to see if this is always true.



Activity B: Dilations and matrices	<u>Get the Gizmo ready:</u> <ul style="list-style-type: none"> • Under Figure type, select Segment. • Turn off the rulers and protractors. 	$\begin{bmatrix} A & B \\ -10 & 2 \\ 0 & 5 \end{bmatrix}$
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1. Set the **Preimage matrix** to $\begin{bmatrix} -10 & 2 \\ 0 & 9 \end{bmatrix}$. Drag the **Scale factor** slider. Look at the matrix equation displayed above the graph as you do.

A. How is the first number related to the scale factor? _____

This number is called a **scalar**.

B. What is the first matrix in the equation the same as? _____

C. How do the elements in the last matrix relate to the coordinates of J and K ? _____

_____ The last matrix is the *image matrix*.

D. How can you use the scalar and the preimage matrix to get the image matrix?

E. Experiment with other points and translations in the Gizmo. How is the image matrix always related to the preimage matrix? _____

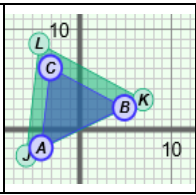
F. What do you think the matrix equation will look like for the dilation of a triangle?

Select **Triangle** from the **Figure type** menu to check.

2. The matrix $\begin{bmatrix} 12 & 4 & -8 \\ 0 & 10 & -6 \end{bmatrix}$ describes the location of the vertices of $\triangle ABC$. In the space

below, write a matrix equation to show the result of dilating $\triangle ABC$ by a scale factor of 0.5. Check your answer in the Gizmo.



<p>Activity C: Using dilations</p>	<p><u>Get the Gizmo ready:</u></p> <ul style="list-style-type: none"> Under Figure type, select Triangle. 	
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1. A triangle has vertices at $A(6, 3)$, $B(8, -4)$, and $C(-2, 5)$. The triangle is dilated and the image of point B lies at $(12, -6)$.

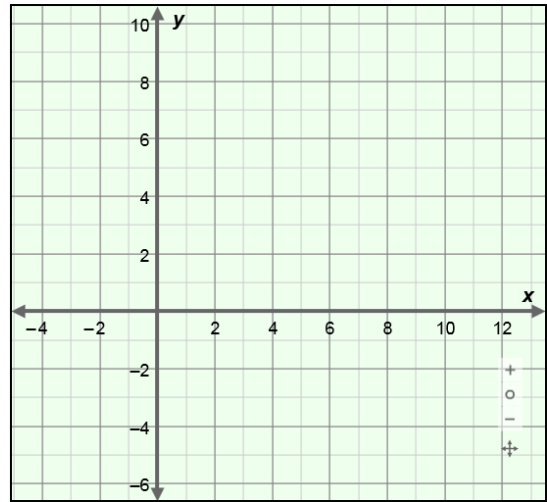
A. Is the image, $\triangle JKL$, larger or smaller than the preimage, $\triangle ABC$? _____

Explain how you know. _____

B. Find the scale factor for this dilation.
Show your work in the space below.

scale factor = _____

Write a matrix equation to find the coordinates of the vertices of $\triangle JKL$.
Show your work in the space below.
Then sketch the preimage and image on the grid to the right. Check your answer in the Gizmo.



2. Andrea is making a copy of a picture that is 8 inches wide and 10 inches tall. The copy is 75% the size of the original picture. Find the dimensions of the copy. Show your work in the space below.

Dimensions of copy: _____ wide and _____ tall

