Name: ______________________________ Date: ____________________

Student Exploration: Direct and Inverse Variation

Vocabulary: constant of proportionality, direct variation, inverse variation

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. Michelle makes $10 an hour babysitting.

   A. How much will she make in 2 hours? ____________ in 4 hours? ____________

   B. How does doubling the babysitting time affect the amount Michelle makes?

   ____

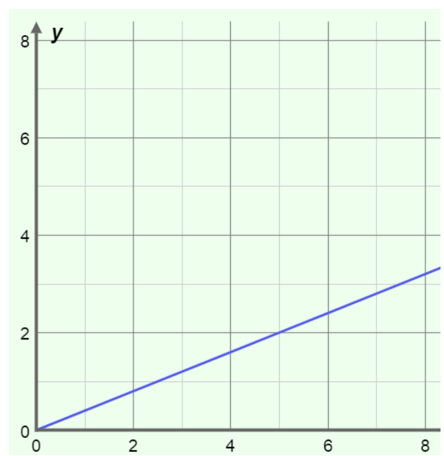
2. A car moving at a speed of 30 miles per hour (mph) will travel 30 miles in one hour.

   A. How long will it take to cover 30 miles at a speed of 60 mph? ______________

   B. How does doubling the speed affect the time? ________________________

Gizmo Warm-up

Earning a certain amount of money per hour is an example of **direct variation** because as the hours increase, the pay increases by a constant factor. The travel time example – where doubling speed causes the time to be cut in half – is an example of **inverse variation**. Situations like these can be modeled in the *Direct and Inverse Variation* Gizmo.



1. On the **CONTROLS** tab, check that **Direct variation** is turned on. Drag the line on the graph. What changes about the line? What stays the same?

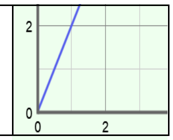
   ____

   ____

2. Drag the ***k*** slider in the Gizmo. How does the graph change as *k* increases?

   ____

   ____

Activity A: **Direct variation**	Get the Gizmo ready: • Check that **Direct variation** is turned on and **Inverse variation** is turned off.	

1. On the **CONTROLS** tab, set ***k*** to 2.0 to graph $y = 2x$. (To quickly set a value, type it in the text field to the right of the slider and hit **Enter**.)

x	y	$\frac{y}{x}$
0.5		
1		
1.5		
2		
2.5		
3		

   A. Substitute the *x*-values shown in the table to the right into the equation to find several points on the line.

   To check your work, select the **TABLE** tab in the Gizmo. Compare your *y*-values with the y_{direct} column in the Gizmo table.

   B. Calculate the values of $\frac{y}{x}$ in the last column of your table. What is true about the ratio of the variables in a direct variation?

   ____

   C. Select the **CONTROLS** tab. How much does *y* change when *x* increases by 1?

   ____

   D. How does the change you described above relate to the slope of the line $y = 2x$?

   ____

   E. In the direct variation equation $y = kx$, k is the **constant of proportionality**. Look at the last column of your table and your answers to the questions above.

   What three things are equal to the value of *k*? _______________________________

   ____

2. In the Gizmo, use different values for *k* in the general equation $y = kx$. Study the resulting graph and table to see what happens to the *y*-value for the following changes in the *x*-value.

   A. If the *x*-value is multiplied by 3, what happens to *y*? _________________________

   B. If the *x*-value is multiplied by 5, what happens to *y*? _________________________

   C. If the *x*-value is divided by 2, what happens to *y*? ___________________________

   D. If the *x*-value is divided by 4, what happens to *y*? ___________________________

(Activity A continued on next page)

Activity A (continued from previous page)

3. If y varies directly as x, then $y = kx$ for some value k.

   A. Suppose y varies directly as x and $y = 20$ when $x = 5$. What is the constant of proportionality, k, in this situation? ___________ Explain how you found k. ________

   ____

   B. In the Gizmo, set ***k*** to the value you found above. Find four other (x, y) pairs that occur in this direct variation function.

   (_____, ______) (_____, ______) (_____, _____) (_____, ______)

4. Suppose you're exchanging money from one form of currency to another, where $1.00 in currency A equals $1.70 in currency B.

   A. If x = amount of currency A and y = amount of currency B, write an equation to model this currency conversion. ____

   B. Graph your equation in the Gizmo. Select the **TABLE** tab. How can you use the table to check if your equation is correct? ____

   ____

   C. How much is $8.00 in currency A equal to in currency B? __________ Explain how you found your answer. ____

   D. How much is $17 in currency B equal to in currency A? ___________ Explain how you found your answer. ____

5. Write an equation to solve each problem. Then solve the problem. Check your answers in the Gizmo.

   A. If y varies directly as x, and $y = 12$ when $x = 2$, find x when $y = 30$.

   equation: ___________ If x = _______ then $y = 30$.

   B. Joe gets paid by the hour. If he earns $54 for working 6 hours, how much will he earn when he works 15 hours?

   equation: ___________ If Joe works 15 hours, he will make ____________

Activity B: **Inverse variation**	Get the Gizmo ready: • On the **CONTROLS** tab, turn off **Direct variation**. • Turn on **Inverse variation**.

1. With the **CONTROLS** tab selected, set ***k*** to 2.0 to graph $y = \frac{2}{x}$.

x	***y***	***xy***
1		
2		
3		
4		
5		
6		
7		
8		

   A. Substitute the *x*-values shown in the table to the right into the equation to find several points on the graph.

   To check your work, select the **TABLE** tab. Set **MIN** to 1.00, **MAX** to 8.00, and **STEP** to 1.00. Compare your *y*-values with the $\boldsymbol{y}_{\textbf{inverse}}$ column of the Gizmo table.

   B. Calculate the product *xy* in the last column of your table. What is true about this product in an inverse variation?

   ____

   C. In the inverse variation equation, *k* is the constant of proportionality. What do you notice about the value of *k* for this equation and the values in the last column of your table?

   ____

   D. Look at the ***x*** and $\boldsymbol{y}_{\textbf{inverse}}$ columns in the Gizmo table. Find the difference in the values of *y* between the following pairs of *x*-values:

   $x = 1$ and $x = 2$ _________ $x = 3$ and $x = 4$ _________ $x = 5$ and $x = 6$ _________

   E. How do the differences above relate to the shape of the graph? ____________________

   ____

2. In the Gizmo, use different values for *k* in the general equation $y = \frac{k}{x}$. Study the resulting graph and table to see what happens to the *y*-value for the following changes in the *x*-value.

   A. If the *x*-value is multiplied by 3, what happens to *y*? ____________________________

   B. If the *x*-value is multiplied by 5, what happens to *y*? ____________________________

   C. If the *x*-value is divided by 2, what happens to *y*? ____________________________

   D. If the *x*-value is divided by 4, what happens to *y*? ____________________________

(Activity B continued on next page)

Activity B (continued from previous page)

3. If y varies inversely as x, then $y = \frac{k}{x}$. Suppose y varies inversely as x and $y = 4$ when $x = 6$.

   A. What is the constant of proportionality, k, in this situation? ______________________

   B. Explain how you found k. ____

   C. On the **CONTROLS** tab, set ***k*** to this value by entering it in the text box and hitting **Enter**. How can the graph show you if this equation is correct? ________________

   ____

4. For anything in motion, distance traveled, d, is equal to the average speed (average rate), r, multiplied by the time traveled, t. This is often abbreviated as $d = rt$.

   A. Suppose you need to drive 75 miles. In the boxes to the right, solve $d = rt$ for time, and use 75 for d. This should give you an equation that shows that time varies inversely with rate.

   $$\boxed{} = \frac{\boxed{}}{\boxed{}}$$

   B. Graph your equation in the Gizmo. (Click the **–** button to the right of the graph to zoom out.) Select the **TABLE** tab. How can the table show you if your equation is correct? ____

   C. How fast would you need to drive to make the trip in 2.5 hours? ________________

   Explain how you found your answer. ____

   ____

5. Daniel's baseball team raised \$90 to buy new baseballs. The team needs to know how many baseballs they can buy, based on how much each one costs.

   A. Write an equation to model this situation. (Hint: Your equation should show that the number of baseballs the team can afford varies inversely with the price of each baseball.)

   $$\boxed{} = \frac{\boxed{}}{\boxed{}}$$

   B. What does each variable in your equation represent? ________________________

   ____

   C. How many baseballs can the team buy if each baseball costs \$2.50? ____________

   Check your answer in the Gizmo.