



Name: _____

Date: _____

Student Exploration: Geometric Sequences

Vocabulary: common ratio, explicit formula, geometric mean, geometric sequence, recursive formula, sequence, term

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

Suppose you are given an allowance of \$1 the first week of the year, and each week your allowance doubles. That is, \$1 the first week, \$2 the second week, \$4 the third week, and so on.

1. If your allowance doubles each week, what is your allowance for the first eight weeks?

_____ \$1 _____, _____ \$2 _____, _____ \$4 _____, _____, _____, _____, _____, _____

2. Starting with \$1 in week 1, how do you calculate your allowances for other weeks?

A. To find week 4 allowance, you have to take \$1 and multiply it by 2 _____ time(s).

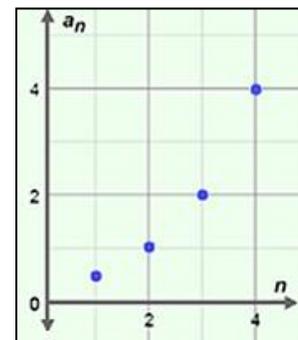
B. To find week 5 allowance, you have to take \$1 and multiply it by 2 _____ time(s).

C. How do you calculate what your allowance will be in week 25? _____

Gizmo Warm-up

A **sequence** is an ordered list of numbers. Each number in a sequence is called a **term**. In a **geometric sequence**, the ratio of any two consecutive terms is constant. The **common ratio** is the ratio of any term and the one before it. In the *Geometric Sequences* Gizmo, you can explore the effects of varying the first term (abbreviated a_1) and the common ratio (r) of a sequence on a graph.

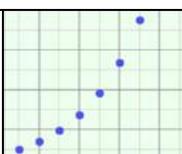
To vary the values of a_1 and r , drag the sliders. To enter a specific value, click on the number in the text field, type the value, and hit **Enter**.



1. In the Gizmo, a sequence is graphed. Vary the first term with the a_1 slider. How does this affect the graph? _____

2. Next vary the common ratio with the r slider. As r increases, what happens to the graph? _____



Activity A: Explicit formula	<u>Get the Gizmo ready:</u> <ul style="list-style-type: none"> • Select the CONTROLS tab. • Unselect all checkboxes. 	
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- Before using the Gizmo, consider a geometric sequence with a first term (a_1) of 0.5 and a common ratio (r) of 2. (Hint: Since $r = 2$, each term will be twice the previous term.)
 - What are the first four terms of the sequence? _____
 - In the Gizmo, set $a_1 = 0.5$ and $r = 2$. The graph in the Gizmo represents the sequence. Click and drag the graph downward to see more points. What are the first four points on this graph? (Place your cursor over any point to see its coordinates.)

 First four points on the graph: _____

 Select the **TABLE** tab. Each row of this table gives the coordinates of a point on the graph. Check your answers, and then return to the **CONTROLS** tab.
 - The points in the graph of a sequence are called (n, a_n) instead of (x, y) .

 For each point, what does n mean? _____

 What does a_n mean? _____
 - What do you think are the coordinates of the 8th point? _____

 Check your answer in the Gizmo, using either the graph or the table.
- Consider the geometric sequence with $a_1 = 8$ and $r = 0.6$. (Do not enter it in the Gizmo yet.) Use a calculator if you like. Note that the Gizmo rounds to the nearest hundredth.
 - What are the first three terms of the sequence? (Round to the nearest hundredth.)

 - What would you multiply the first term by to find the 6th term? _____
 - What can the first term be multiplied by to find the n th term? _____

 An **explicit formula** is a rule allowing direct calculation of any term in the sequence. The explicit formula for the n th term of an geometric sequence is $a_n = a_1 \cdot r^{n-1}$.
 - In the Gizmo, set $a_1 = 8$ and $r = 0.6$ and turn on **Show explicit formula**. Use the explicit formula to find a_7 . Show your work in the space to the right. Then check your answer in the Gizmo.

(Activity A continued on next page)



Activity A (continued from previous page)

3. In the Gizmo, graph the geometric sequence with $a_1 = 2$ and $r = -3$.

A. What are the first four terms? _____

B. How did the negative value of r affect the terms of the sequence? _____

Explain. _____

C. How would the graph look if $r = 1$? _____

Explain. _____

Use the Gizmo to check your answer.

4. Before using the Gizmo, consider the geometric sequence with $a_1 = 4$ and $r = 0.5$.

A. Write the explicit formula for the n th term of the sequence. _____

B. In the space to the right, use the explicit formula to find the value of a_5 .

Enter the values of a_1 and r in the Gizmo and check your answer.

C. If $a_1 = -4$, how would the graph change? _____

D. What do you think is the value of a_5 of this new sequence? _____

Check your answers in the Gizmo.

5. A geometric sequence is graphed to the right.

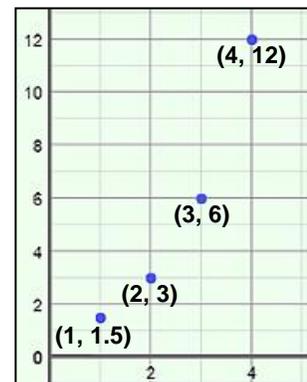
A. What are the first four terms? _____

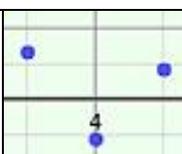
B. What is a_1 ? _____ What is r ? _____

C. Write the explicit formula for a_n . _____

D. What is a_7 ? _____
Show your work to the right.

Then check your answer in the Gizmo.



Activity B: Recursive formula	<u>Get the Gizmo ready:</u> <ul style="list-style-type: none"> • Be sure the CONTROLS tab is selected. • Select Show explicit formula. 	
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1. Consider the geometric sequence 3, 6, 12, 24, ... (Do not enter it in the Gizmo yet.)

A. What is the value of a_1 ? _____ What is the value of r ? _____

B. What are the next three terms of the sequence? _____

Explain. _____

C. What is the explicit formula for the n th term? _____

Enter a_1 and r in the Gizmo. Check your answers and make necessary corrections.

D. Use the explicit formula to find a_8 . Show your work in the space to the right. Then check your answer in the Gizmo.

E. In this sequence, the 16th term is 98,304. What is the 17th term? _____

Explain. _____

F. If the 16th term is 98,304, what is the 15th term? _____

Explain. _____

G. For this sequence, how can you find the n th term, a_n , if you know the previous term?

A **recursive formula** is a rule for finding a term in a sequence based on the previous term. In general, for a geometric sequence, the recursive formula is $a_n = a_{n-1} \cdot r$. That rule plus the value of the first term (a_1) defines the sequence.

2. Before using the Gizmo, consider the sequence defined by $a_1 = -8$ and $r = 0.5$.

A. What are the first four terms of the sequence? _____

B. Fill in the recursive part of the rule for this sequence: $a_1 = -8$, $a_n =$ _____

C. In the space to the right, find a_8 . Show your work, and round to the nearest hundredth. Then check your answer in the Gizmo.

(Activity B continued on next page)

Activity B (continued from previous page)

3. Consider the geometric sequence $-0.5, -1, -2, -4, \dots$

A. Express this sequence both explicitly and recursively.

Explicit: _____ Recursive: _____

B. What is a_{10} ? _____ Check your answer in the Gizmo.

C. Which formula works better for finding terms that are later in the sequence, like a_{20} ?

_____ Explain why. _____

4. In a geometric sequence, $a_3 = 100$ and $a_5 = 2500$.

A. What is r ? _____ Explain. _____

B. What is a_4 ? _____

C. If you take the square root of the product of a_3 and a_5 , what is the result? _____

How does this compare to a_4 ? _____

The **geometric mean** of two numbers is the square root of their product. In the sequence above, a_4 is the geometric mean of a_3 and a_5 .

5. The graph to the right shows the first four terms of a geometric sequence.

A. What is a_1 ? _____ What is r ? _____

B. Write the recursive formula for the sequence.

C. What is the geometric mean of 2 and 4.5? _____

Explain. _____

