Name: _____     Date: _____

# Student Exploration: Solving Linear Systems (Matrices and Special Solutions)

**Vocabulary:** consistent system, dependent system, determinant, inconsistent system, independent system, matrix, solution, system of linear equations

**Prior Knowledge Questions** (Do these BEFORE using the Gizmo.)

1. The sum of Max's age ($x$) and his sister's age ($y$) is 8.

   A. What could their ages be? _____

   B. Write an equation for this situation. _____

2. The difference of Mary's age ($y$) and her older brother's age ($x$) is 2.

   A. What could their ages be? _____

   B. Write an equation for this situation. _____

3. Suppose Max is Mary's older brother. How old are they? _____

**Gizmo Warm-up**

The equations $x + y = 8$ and $x - y = 2$ form a **system of linear equations** in standard form. In the *Solving Linear Systems (Matrices and Special Solutions)* Gizmo, you can use graphing or algebra to solve such systems.

On the **CONTROLS** tab, select the top equation. Set **A** to 1, **B** to 1, and **C** to 8 to graph $x + y = 8$. Select the bottom equation. Set **A** to 1, **B** to –1, and **C** to 2 to graph $x - y = 2$. (To quickly set the value of a slider, type the number into the text box to the right of the slider and press **Enter**.)
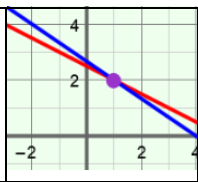
| CONTROLS | MATRIX FORM | TABLE |
| --- | --- | --- |

○ $x + y = 8$ 🟥
◉ $x - y = 2$ 🟦

A ———————●——— 1
B ————●——————— -1
C ——————————●— 2

☐ Check solution at point

1. Look at the graph. How many points of intersection do the lines have? _____

2. Mouseover the purple intersection point. What are the coordinates?   (_____, _____)

3. How do these values relate to Max and Mary's ages? _____

| **Activity A:** | Get the Gizmo ready: |
|---|---|
| **Solutions of systems** | • On the **CONTROLS** tab, select **Check solution at point**. |



1. Jack buys 1 snowdrop and 2 chocomalts for a total cost of $5. Jill buys 2 snowdrops and 3 chocomalts for a total cost of $8.

    A. Write a system of equations that describes their purchases. Use *x* for the cost of a snowdrop and *y* for the cost of a chocomalt. Graph this system in the Gizmo.

        _____

    B. Mouseover the purple point. What are the coordinates?  (_____, _____)

    C. What do you think will happen if you substitute these coordinates into each equation

       and then simplify? _____

    D. In the space to the right, substitute the coordinates of the intersection point into each equation and simplify.

       Is each equation true? _____

       Drag the green point to the intersection point to check. The coordinates of this point are the **solution** of the system because they make both equations true.

    E. Do you think there are other (*x*, *y*) pairs that are solutions of this system? _____

       Why or why not? _____

       _____

       Select **Show number of solutions** to check your answer. Systems of equations with exactly one solution are **consistent** and **independent**.

    F. What is the cost of each snowdrop and each chocomalt?

         cost of snowdrop = _____        cost of chocomalt = _____

    G. Notice that the lines for this system have different slopes. In the Gizmo, experiment with more systems whose lines have different slopes. What do you notice?

       _____

**(Activity A continued on next page)**

**Activity A (continued from previous page)**

2. You should have seen that a system of two lines with different slopes has one solution.

   A. Suppose the graph of a system is two lines with the same slope but different

   *y*-intercepts. What do you think the solution of this system is? _____

   Explain. _____

   B. Suppose the graph of a system is two identical lines. What do you think the solution

   of this system is? _____

   Explain. _____

3. On the **CONTROLS** tab, turn off **Check solution at point** and **Show number of solutions**.
   Consider the system of equations $2x + 2y = 4$ and $x + y = 2$.

   A. Multiply each side of $x + y = 2$ by 2. What is true about the equations now?

   _____

   B. What do you think the graph of this system will look like? _____

   Graph the system in the Gizmo to check your answer.

   C. How many solutions does this system have? _____

   Select **Show number of solutions** to check your answer. This system is **consistent**
   (because it has at least one solution) and **dependent** (because it has an infinite
   number of solutions).

   D. Experiment with other systems of equations in which one equation is a multiple of the

   other. What do you notice about the solutions? _____

4. Turn off **Show number of solutions**. In the Gizmo, graph $x - y = 2$ and $3x - 3y = -8$.

   A. What is true about the lines in the graph? _____

   B. How many solutions does this system have? _____

   Select **Show number of solutions** to check your answer. This system is
   **inconsistent** because it does not have a solution.

   C. What would you change in the second equation to make this a dependent system?

   _____

| Activity B:<br><br>**Matrices and linear systems** | Get the Gizmo ready:<br><br>• On the **CONTROLS** tab, turn on **Check solution at point** and turn off **Show number of solutions**. | $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ |
|---|---|---|

1.  Graph the system $2x + y = -2$ and $3x + 4y = 7$ in the Gizmo. What is the solution of this

    system?  (_____, _____)  Drag the green point to the purple point to check.

2.  A system of equations can be written using **matrices** (rectangular arrays of numbers). To see how $2x + y = -2$ and $3x + 4y = 7$ is written in matrix form, select the **MATRIX FORM** tab.

    A.  The matrix equation appears under the system of equations. Write the matrix equation to the right.

    B.  How is the left matrix related to the equations? _____

        _____  This is the *coefficient matrix*.

    C.  How is the right matrix related to the equations? _____

3.  You can use the **determinant** of the coefficient matrix to help find the number of solutions of a system. To calculate the determinant, find the difference of the products of the diagonal elements as shown to the right.    $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

    A.  Graph each system below and determine the number of solutions. Select **Show number of solutions** to check. Then calculate the determinant of the coefficient matrix. On the **MATRIX FORM** tab, select **Show determinant** to check.

| System | Slopes – same or different? | Number of solutions | Value of determinant |
|---|---|---|---|
| $2x + y = 3$<br>$x + 2y = 6$ | | | |
| $4x - 4y = 2$<br>$2x - 2y = 1$ | | | |
| $3x + y = 1$<br>$3x + y = -2$ | | | |

    B.  Try a variety of systems. How does the determinant of the coefficient matrix relate to

        the slopes of the lines and the number of solutions? _____

        _____

        _____

**(Activity B continued on next page)**

**Activity B (continued from previous page)**

4. Write the matrix equation for each system below. Then find the determinant of the coefficient matrix, and state how many solutions the system has. Check your answers in the Gizmo.

A. $x + y = 2$
$x - y = 6$

C. $5x + y = 4$
$5x + y = -6$

Number of solutions: _____

Number of solutions: _____

B. $2x + y = 3$
$4x + 2y = 6$

D. $3x + 2y = -1$
$3x + y = 4$

Number of solutions: _____

Number of solutions: _____

5. Write a system of linear equations in standard form for each number of solutions below. Find the determinant of the coefficient matrix of each system. Check your answers in the Gizmo.

| Number of solutions | System of linear equations | Value of determinant |
|---|---|---|
| one | | |
| infinitely many | | |
| none | | |