Name: _____ Date: _____

Student Exploration: Logarithmic Functions: Translating and Scaling

Vocabulary: asymptote, base, domain, logarithmic function, scale (a function), transform (a function), translate (a function)

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. The function $y = 3^x$ is an exponential function, because the variable is in the exponent. The value (in this function, the 3) raised to the exponent is called the **base** of the function.

   A. For $y = 3^x$, what is the output (y) when the input (x) is 2? _____

   B. What is y when x is 5?_____

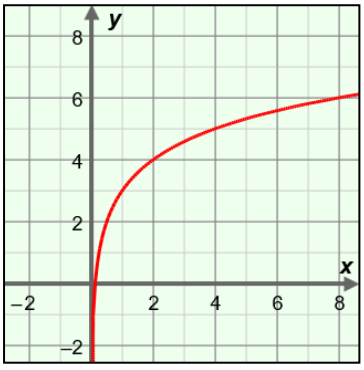
2. The inverse of $y = 3^x$ is the **logarithmic function** $y = \log_3(x)$. Recall that the logarithmic form ($y = \log_3(x)$) can always be converted to exponential form ($3^y = x$) if it helps.

   A. For $y = \log_3(x)$, what does the *input* (x) have to be to get an *output* (y) of 2? _____

   B. For $y = \log_3(x)$, what is x when y is 5? _____

Gizmo Warm-up

The general form of a logarithmic function is $y = a \log_b c(x – h) + k$. In the *Logarithmic Functions: Translating and Scaling* Gizmo, you can vary the values of a, c, h, and k to see how they alter, or **transform**, the graph of $y = \log_b(x)$.



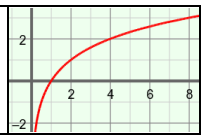
On the **CONTROLS** tab, be sure ***a*** is set to 1, ***b*** to 2, ***c*** to 1, ***h*** to 0, and ***k*** to 0. (To quickly set the value of a slider, type the number into the text box and press **Enter**.) The function graphed in the Gizmo should be $y = \log_2(x)$.

1. Vary ***h*** by dragging the slider. What happens to the graph? _____

   _____

2. Drag the ***k*** slider back and forth. How does the graph change as you vary k? _____

   _____

Activity A: **Effects of *h* and *k* on the graph**	Get the Gizmo ready: • On the **CONTROLS** tab, set ***a*** to 1, ***b*** to 2, ***c*** to 1, ***h*** to 0, and ***k*** to 0.	

1. The function you have graphed in the Gizmo should be $y = \log_2(x)$.

   A. Give the coordinates of two "key points" on the graph of $y = \log_2(x)$ (the points that have *y*-values of 0 and 1). (____, 0) and (____, 1) Select **Show probe** to check.

   B. Set ***k*** to 3 to graph the function $y = \log_2(x) + 3$. What are the coordinates of these two key points now? (_____, _____) (_____, _____)

   C. How did adding 3 to $\log_2(x)$ change the coordinates of those points? _____

   _____

   D. How did adding 3 **translate** (shift) the graph as a whole? _____

   _____

   E. Vary ***k*** and ***b*** to see how *k* affects logarithmic functions with different bases. In general, what two key points are always on the graph of $y = \log_b(x) + k$?

   (1, _____) and (*b*, _____)

2. Graph $y = \log_2(x)$ again in the Gizmo. Then set ***h*** to 3 to graph the function $y = \log_2(x - 3)$.

   A. Where are the key points (formerly (1, 0) and (2, 1)) now? (___, ____) and (___, ____)

   B. How did subtracting 3 from *x* shift the graph of $y = \log_2(x)$? _____

   Explain why this makes sense. _____

   _____

   _____

   C. Vary ***h*** and ***b*** to see how *h* affects logarithmic functions with different bases. In general, what two key points are always on the graph of $y = \log_b(x - h)$?

   (_____, 0) and (_____, 1)

   D. In general, how do you know which direction *h* will move a graph? _____

   _____

(Activity A continued on next page)

Activity A (continued from previous page)

3. Experiment more with different values of ***h***, ***k***, and ***b***. In general, what two key points are always on the graph of $y = \log_b(x - h) + k$? (_____, _____) (_____, _____)

4. Now graph $y = \log_8(x)$ in the Gizmo. Turn on **Show asymptote**. The light blue vertical dotted line is the **asymptote**. It marks the edge of the **domain**, which is the set of all x-values for which the function is defined.

   A. How do you think h and k affect the asymptote and domain of $y = \log_8(x)$? Explain.

   _____

   _____

   B. Vary ***h*** and ***k*** in the Gizmo to check your answer. What do you find?

   _____

   _____

   C. Vary ***b***, ***h***, and ***k*** to test many different logarithmic functions. In general, what are the asymptote and domain of $y = \log_b(x - h) + k$?

   Asymptote: _____ Domain: _____

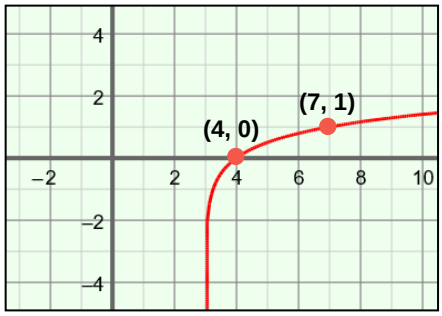
5. State the coordinates of two key points on the graphs of the following functions. Check your answer in the Gizmo.

   A. $y = \log_2(x - 5)$ B. $y = \log_6(x) + 2$ C. $y = \log_3(x + 4) - 6$

   _____ _____ _____

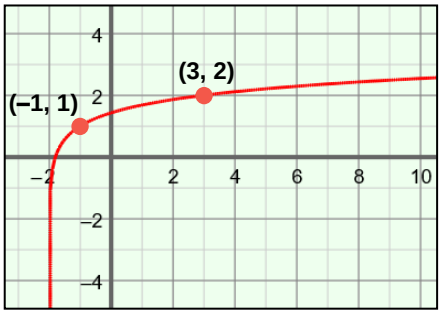
6. What logarithmic functions are graphed here? Check your answers in the Gizmo.

   A.

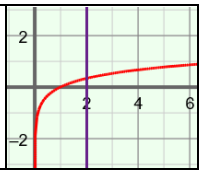


   _____

   B.



   _____

Activity B: **Effects of *a* and *c* on the graph**	Get the Gizmo ready: • On the **CONTROLS** tab, select **Show probe** and turn off **Show asymptote**. • Set ***a*** to 1, ***b*** to 8, ***c*** to 1, ***h*** to 0, and ***k*** to 0.	

1. The function you have graphed in the Gizmo should be $y = \log_8(x)$.

   A. What are the coordinates of two "key points" (the points with *y*-values of 0 and 1) on this graph? (_____, 0) and (_____, 1) Use the probe to check your answers.

   B. Use the slider to vary the value of ***a***. What happens to these two points? _____

   _____

   C. When *a* is negative, what happens to the graph? _____

   _____

   D. Vary ***a*** and ***b*** to check how *a* affects logarithmic functions with other bases. You should see that the value of *a* **scales** (stretches or shrinks) the graph vertically.

   In general, what two key points are always on the graph of $y = a \log_b(x)$?

   (_____, _____) and (_____, _____)

   E. Select **Show asymptote**. Vary ***a*** and ***b***. How does *a* affect the asymptote and domain? _____

2. Graph the function $y = \log_5(x)$ in the Gizmo.

   A. What is the solution to $\log_5(x) = 0$? _____ To $\log_5(x) = 1$? _____

   B. Based on that, what two points lie on the graph of $y = \log_5(x)$? _____

   C. What is the solution to $\log_5 2(x) = 0$? _____ To $\log_5 2(x) = 1$? _____

   D. So, what two points must lie on the graph of $y = \log_5 2(x)$? _____

   E. Graph $y = \log_5 2(x)$ in the Gizmo. Use the probe to verify the points you found. Overall, how did the graph of $y = \log_5(x)$ change when *c* changed from 1 to 2?

   _____

   _____

(Activity B continued on next page)

Activity B (continued from previous page)

3. Vary ***c*** and ***b*** to check how *c* affects logarithmic functions with other bases. You should see that *c* scales the graph horizontally.

   What two key points are always on the graph of $y = \log_b c(x)$? _____

4. Graph $y = \log_2(x)$ in the Gizmo. Turn on **Show asymptote**.

   A. How do you think *a* and *c* will affect the asymptote and domain of $y = \log_2(x)$?

   _____

   _____

   Check your answer in the Gizmo.

   B. Vary ***a*** and ***c*** in the Gizmo to check your answer. What do you find?

   _____

   _____

   C. Vary ***a***, ***b***, and ***c***, to see many different logarithmic functions. In general, what are the asymptote and domain of $y = a \log_b c(x)$?

   Asymptote: _____ Domain: _____

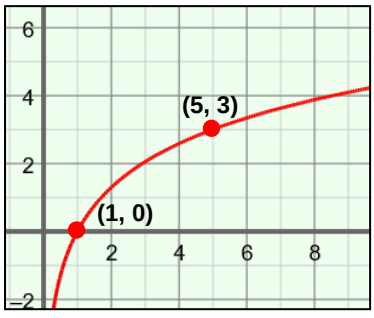
5. State the coordinates of two key points on the graphs of the following functions. Check your answers in the Gizmo.

   A. $y = 6 \log_3(x)$ B. $y = \log_6 2(x)$ C. $y = -5 \log_9 2(x)$

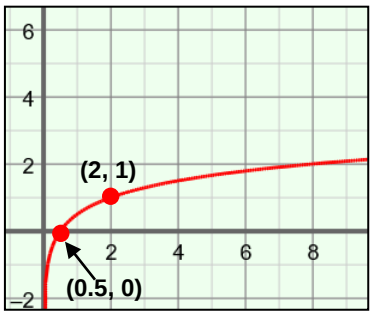
   _____ _____ _____

6. What logarithmic functions are graphed here? Check your answers in the Gizmo.

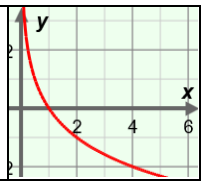
   A.



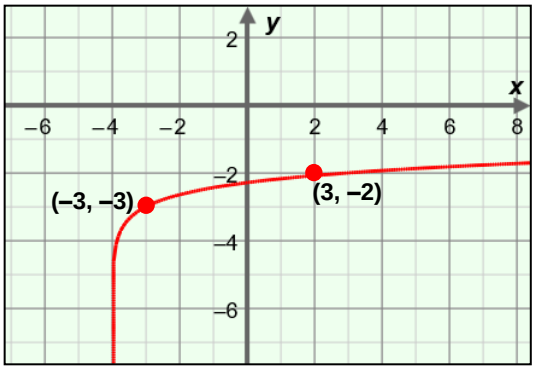
   B.



   _____ _____

Activity C: **Practice scaling and translating functions**	Get the Gizmo ready: • On the **CONTROLS** tab, turn off **Show probe** and **Show asymptote**.	

1. Before using the Gizmo, consider the graph shown here.



   A. How has the function $y = \log_7(x)$ been transformed to create the graph shown?

   _____

   _____

   _____

   B. What is the value of h? _____ Of k? _____

   C. What logarithmic function does the graph show? _____

   Use the Gizmo to check your answers.

2. State the coordinates of two key points on the graphs of the following functions. (Hint: Work from the "inside out.")

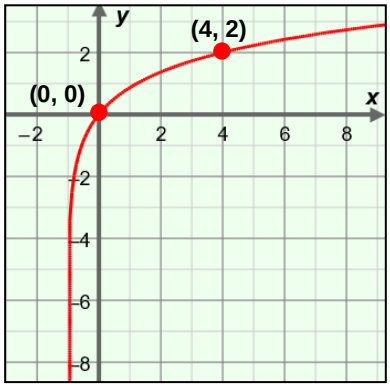
   A. $y = 7 \log_3(x)$ B. $y = -\log_8(x + 2) + 5$ C. $y = \log_4 2(x) - 6$

   _____ _____ _____

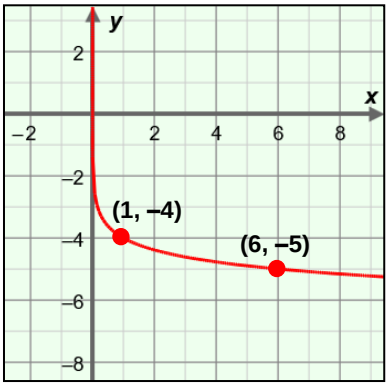
   Graph these functions in the Gizmo to check your answers.

3. What logarithmic functions are graphed here? Check your answers in the Gizmo.

   A.



   B.



   _____ _____