Name: _____   Date: _____

Student Exploration: Lucky Duck

Vocabulary: expected value, experimental probability, fair game, outcome, probability distribution, random variable, theoretical probability

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
Arnie the carnie, from behind his carnival booth, catches you on the way to the Ferris wheel. "Hey there! Think you can draw the lucky duck? Don't let your chance float away!"

1. Arnie's game has 10 rubber ducks in a pond, each with a prize on the bottom. The ducks look identical, but 2 of them win you a big prize, 3 win a small prize, and 5 win no prize at all.

    If you pick one duck at random, what is the probability of winning a small prize? _____

2. After a player picks a duck, Arnie always puts that duck back in the pond. How does this affect the next player's chances to win?

    _____

Gizmo Warm-up
In the *Lucky Duck* Gizmo™, you draw ducks at a carnival game. You win the prize shown on the bottom of the duck. See if you can beat the odds! More importantly, see if you can help your friend Arnie the carnie break even, or make money.

1. In the Gizmo, click on a duck to draw it (or click **Draw 1**).

    What did you win? _____

    The prize you win varies, and is random, so it is an example of a **random variable**.

2. Click **Clear**. Press the **Draw 10** button. How many of each type of prize did you win?

    No prize _____   Small prize (banana) _____   Big prize (monkey) _____

    The result of each draw is called an **outcome** of the experiment.

| **Activity A:**

What happens in the long run? | Get the Gizmo ready:

 • In **Play the Game** mode, be sure **Game A** is selected and the game cost is $1.50.
 • If not, click refresh or reload in your browser.
 • Click **Clear** to erase any old results. | |

You are back at your favorite carnival game, Lucky Duck, run by your friend Arnie the carnie. Today's prizes are a stuffed banana (small prize) and a stuffed monkey (big prize).

1. You step up to play. Click **Draw 1** (or click on a duck). What did you win? _____

2. Curious about today's odds, you decide to watch a few more people play the game.

   A. Click **Draw 10**. Now that 11 people (including you) have played, how many won:

   No prize _____   Small prize _____   Big prize _____

   B. Now write the frequency of each outcome as a *fraction* (out of 11).

   No prize _____   Small prize _____   Big prize _____

   Each fraction gives the **experimental probability** of that outcome.

   C. By the end of the day, 100 more people have played Lucky Duck (for a total of 111). Click **Draw 100**. Based on your results now, what are the experimental probabilities?

   No prize _____   Small prize _____   Big prize _____

   D. Why are "end-of-the-day" estimates probably better than early ones? _____

   _____

3. As night falls, Arnie closes Lucky Duck, after 111 people played. He asks for your help figuring out if he made or lost money today. Fill in the values below:

   A. Arnie's revenue per player _____   Total revenue today _____

   B. How much did Arnie pay for each prize? (Click or tap a prize to see its value.)

   Arnie paid _____ for each small prize and _____ for each big prize.

   C. What were his total costs (total giveaway value) and avg. giveaway per player today?

   Total giveaway value _____   Avg. giveaway value (per player) _____

   D. What is his net gain/loss today? _____   Click **Show cost/revenue** to check.

(Activity A continued on next page)

Activity A (continued from previous page)

4. Arnie has an app that graphs his data over the course of that day. Click the **Trends** tablet (or click **Show trends**). Zoom out (**–**) a few times to see today's **Net gain/loss** graph.

    A. What does today's net gain/loss graph look like? _____

    _____

    B. The next day, Arnie has 100 more people play his game. Click **Clear** and **Draw 100**. How does today's net gain/loss graph compare to yesterday's graph?

    _____

5. Go to **Design the game** mode. This shows you the actual numbers of ducks Arnie has in the pond. Fill the table with the values from the Gizmo.

	No prize	**Small prize**	**Big prize**
Number of ducks			
Prize value	$0		

    A. How likely is each outcome when drawing one duck? (Write answers as fractions.)

    No prize _____   Small prize _____   Big prize _____

    These fractions give the **theoretical probability** of each outcome.

    B. Theoretically, if 100 customers played Lucky Duck one day with the current setup (**Game A**), and the results exactly matched the probabilities, how many would win:

    No prize _____   Small prize _____   Big prize _____

    C. If a small prize costs Arnie $2, and a large prize costs $8, what would the total value of prizes given away to the 100 customers be? _____

    D. For these 100 games, what would the average prize giveaway value be? _____

    The **expected value** is the long-run average outcome of a random variable.

    E. Why is Arnie losing money with his current game setup? _____

    _____

    F. If Arnie wants to break even or make money, what would you advise him to do?

    _____

| Activity B:

Fair games | Get the Gizmo ready:

• Be sure **Game A** is selected, and the cost to play is $1.50. (If not, refresh or reload your browser.)
• In **Play the game** mode, click **Clear**. | |

1. Arnie the carnie runs Lucky Duck. He's been losing money, and asks for your help. He tells you he uses the **Game A** setup, charges $1.50 to play, and gets about 100 players per day.

   A. You ask for a few days of data. Open the **Cost/revenue** table and **Trends** graphs. Click **Draw 100**. Record Arnie's net gain/loss and average giveaway for day 1.

	Players	Net gain/loss	Avg. giveaway	Cost to play
Day 1	100			$1.50
Day 2	100			$1.50
Day 3	100			$1.50

   B. Click **Clear** and repeat the process for days 2 and 3. Fill in the values in the table.

   C. Why is Arnie losing money? _____

   _____

2. Arnie agrees to show you how many ducks of each type he actually has in the pond.

   A. Select **Design the game**. Use the numbers there to fill in this table.

	No prize	Small prize	Big prize
Theoretical probability (as a fraction)			
Prize value	$0		

   B. Draw a bar graph that displays these probabilities, for the three prize values used in **Game A**.

      When you are done, click **Show probability distribution** to check your bar graph in the Gizmo.

      A **probability distribution** is a table or graph that lists all the possible outcomes and probabilities of a random variable.

(Activity B continued on next page)

Activity B (continued from previous page)

3. Now that Arnie has shown you his **Game A** setup (numbers of ducks and prize values), you tell him that you'll go run some numbers for him.

    A. In the given space, find the expected value of Arnie's average giveaway.

       Select **Show expected value** to see if your answer matches the carnival bookkeeper's answer.

    B. How much should Arnie charge for this game, to break even? _____

    C. Set your game cost in the Gizmo. Then, in **Play the game** mode, click **Draw 100** several times. How is Arnie doing now?

         _____

       If Arnie's long-term average gain/loss is zero, then from a player's perspective, the cost to play equals their expected winnings. That is the definition of a **fair game**.

4. Arnie would like to try two new games setups later this week – **Game B** and **Game C**.

    A. Figure out what Arnie should charge to make **Game B** and **Game C** fair games. Show your work in the space given here. Then check your answers in the Gizmo.

       Fair cost for **Game B**: _____   Fair cost for **Game C**: _____

    B. Play **Game C** with the fair cost you just found, for 5 days (100 players per day). Record Arnie's gain or loss for all 5 days. (Click **Clear** at the start of each day.)

GAME C	Day 1	Day 2	Day 3	Day 4	Day 5
Gain/Loss					

    C. Arnie calls you, a bit annoyed. He says he always sets up a fair game, but still loses money some days. He thinks something must be wrong. What would you tell him?

        _____

        _____

Activity C:	Get the Gizmo ready:	
Lucky Duck game master	• Select **Play the game**. • Click **Clear**.	

Arnie is ready to have a business partner for his Lucky Duck game. He wants to hire you as his Lucky Duck assistant and game master, but first he wants to make sure you know your stuff.

1. Arnie sets up a random game of Lucky Duck. (Select **Random** from the **Game** dropdown.) He asks you to draw ducks until you think you know what cost will make this a fair game.

   A. What game cost do you estimate will make this game fair? _____

   B. How many ducks did you draw to feel confident in your estimate? Explain. _____

   _____

   C. In **Design the game**, click **Show expected value**. How close was your estimate?

   _____

2. Next weekend, Arnie wants to charge $3.00 per draw on both Saturday and Sunday, but with different game setups (prize values and numbers of ducks) each day.

   A. In **Design the game**, create two different setups for Arnie. Be sure they are fair games with a $3.00 cost. Write the values for each setup in the tables.

SATURDAY	No prize	Small prize	Big prize
Number of ducks			
Prize value	$0		

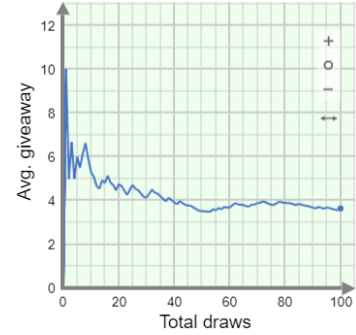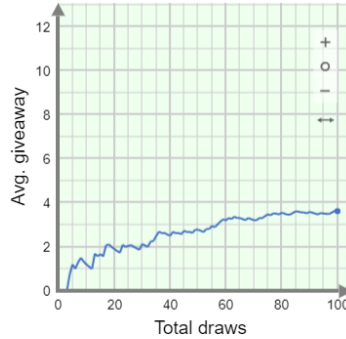| SUNDAY	No prize	Small prize	Big prize
Number of ducks			
Prize value	$0		

   B. In this space, show Arnie why $3.00 is a fair price for both setups. Then, in the Gizmo, play each game for at least 100 draws. Click **Show expected value** for both.

(Activity C continued on next page)

Activity C (continued from previous page)

3. Arnie wants help interpreting the graphs in his app. These two graphs show his average giveaway value for two different days. He used the same setup (ducks and prize values) on both days.

A. Explain to Arnie how these graphs could happen, even with the exact same setup.

_____

_____

B. Why do both graphs flatten out? _____

_____

C. Roughly what game cost would make this a fair game? _____

4. Just for fun, Arnie asks you to design games with the **biggest** and **smallest** possible expected giveaway values. Enter your game setups in the Gizmo, and in the tables here.

BIGGEST	No prize	Small prize	Big prize
Number of ducks			
Prize value	$0		

SMALLEST	No prize	Small prize	Big prize
Number of ducks			
Prize value	$0		

What are the expected average giveaway values for these setups? Calculate them in the space given here. Then check them by playing your games in the Gizmo.

Expected average giveaway values: Biggest _____   Smallest _____