Gizmos

Name: ______________________________________ Date: ________________________

Student Exploration: Moment of Inertia

Vocabulary: angular velocity, linear velocity, moment of inertia, rotational kinetic energy, translational kinetic energy

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
At the finale of her routine, a figure skater starts to spin slowly in the middle of the ice rink, her arms and legs artfully outstretched.

1. What will happen as she stands up straight and pulls her arms in?

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2. Why do you think this will happen? ____________________________

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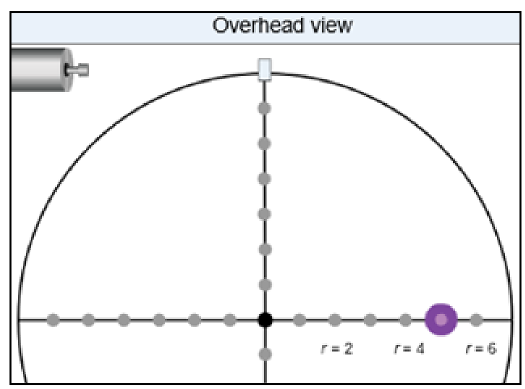
Gizmo Warm-up
The *Moment of Inertia* Gizmo allows you to explore the factors that affect how quickly objects spin. The Gizmo shows a weightless turntable with several pegs. You can place the purple 1-kg masses on any of the pegs. When the Gizmo opens, there is a single 1-kg mass located 5 m from the center.



1. Click **Play** (▶). What happens? __________

   ____

2. The speed of rotation is described by **angular velocity** (ω), which is measured in radians per second. There are 2π radians (6.28 radians) in a circle.

   What is the current angular velocity? ____________________

3. Click **Reset** (↺). Move the mass so that it is only 1 meter from the center (r = 1).

   A. Click **Play**. What is the angular velocity now? ____________________

   B. How is this situation similar to a spinning skater? _________________________

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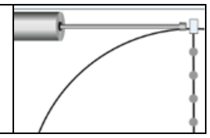
Activity A: **Angular velocity**	Get the Gizmo ready: • Click **Reset**. • Check that the **Piston energy** is set to 200 J.	

Introduction: For a spinning turntable, two types of velocity are important. As you learned, the angular velocity (*ω*) describes how quickly the whole disk is spinning in radians per second. An angular velocity of 6.28 rad/s describes a disk that makes one full revolution every second.

You can also describe the **linear velocity** (*v*) of a point on the turntable. Linear velocity is simply the speed and direction of a point at an instant in time. Linear velocity is measured in meters per second.

Question: How can you predict angular velocity based on the total energy of the turntable and how its mass is distributed?

1. Explore: Experiment with the Gizmo to answer the following two questions:

   A. How does increasing the mass of the turntable affect its angular velocity? __________

   ____

   B. How does increasing the average distance of the masses from the center affect the angular velocity of the turntable? ____

   ____

2. Gather data: Turn on **Show linear velocity**. The Gizmo reports the velocity of the red dot.

   Place the red dot at 1 meter and a single mass at 5 meters. Click **Play**. For each position of the red dot, record the angular velocity of the turntable and linear velocity of the red dot.

Red dot position:	*r* = 1 m	*r* = 2 m	*r* = 3 m	*r* = 4 m	*r* = 5 m	*r* = 6 m
Angular velocity:						
Linear velocity:						

3. Analyze: How does the linear velocity relate to the angular velocity and the radius? _______

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   Write an equation for the linear velocity (*v*) based on angular velocity (*ω*) and position of the red dot (*r*):

   v =

(Activity A continued on next page)

Activity A (continued from previous page)

4. Gather data: Place a single mass on the turntable at a radius of 6 m, and place the red dot on the mass. Click **Play**, and record both the angular velocity (ω) of the turntable and the linear velocity (v) of the mass. Move the mass *and* red dot to the next radius and repeat.

Radius (m)	Angular velocity (rad/s)	Linear velocity (m/s)	Kinetic energy (J)
6 m			
3 m			
2 m			

5. Analyze: What patterns do you notice in your data? ________________________________

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6. Calculate: The **translational kinetic energy** of a rotating object is equal to one half its mass multiplied by its velocity squared: $TKE = ½ mv^2$. Use the linear velocity to calculate the last column of the table. (Hint: There is one mass on the turntable, so its mass is 1 kg.)

   A. How does the translational kinetic energy of the turntable compare to the piston

      energy? ____

   B. Why do you think this is so? ____

      ____

7. Manipulate: Suppose a weightless turntable contains a single mass m that is a distance r from the center. The turntable has a kinetic energy KE.

   A. How would you calculate the linear velocity of the mass? v =

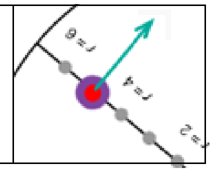
   B. How would you calculate the angular velocity of the turntable? ω =

   C. Given the radius r, the mass m, and the kinetic energy KE, what is the angular velocity of the turntable? ω =

8. Apply: A weightless turntable has a mass of 4 kg that is located 3 m from the center. It is struck by a piston that imparts 160 J of kinetic energy to the turntable.

   What is the angular velocity of the turntable? ____________________

   Use the Gizmo to check your answer. (Hint: Place a mass on each of the 3-m pegs.)

Activity B: **Moment of inertia**	Get the Gizmo ready: • Click **Reset**. Turn off **Show linear velocity**. • Set the **Piston energy** to 200 J. • Place a single mass at *r* = 5 m.	

Introduction: When studying more complex rotating objects, it is helpful to use the rotational equivalent of mass, **moment of inertia** (I). Just as mass can be defined as the resistance to changes in linear velocity, moment of inertia is a measure of an object's resistance to changes in angular velocity.

Question: How is angular velocity related to kinetic energy and moment of inertia?

1. Compare: The **rotational kinetic energy** of a body is given by the formula $RKE = ½ I\omega^2$.

   How is this equation similar to the equation for translational kinetic energy? _____________

   ____

2. Calculate: Click **Play**. Calculate the value of I based on $RKE = ½ I\omega^2$, the current energy value (200 J), and the angular velocity (ω). The units of I are kg·m^2.

   I = ____________________ Turn on **Show moment of inertia** to check your answer.

3. Explore: Place the 1-kg mass at each radius and record the moment of inertia.

Position:	*r* = 1 m	*r* = 2 m	*r* = 3 m	*r* = 4 m	*r* = 5 m	*r* = 6 m
Moment of inertia:						

4. Analyze: How is the moment of inertia related to the radius? _________________________

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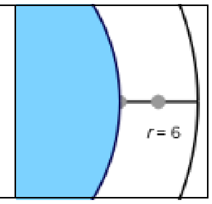
5. Practice: The moment of inertia for a single object is given by the formula $I = mr^2$. When there are multiple objects, the total moment of inertia is the sum of the individual moments.

   A. What is the moment of inertia for a system that has a 2-kg mass that is 2 meters from the center and a 3-kg mass that is 3 meters from the center? ______________

   B. Suppose the kinetic energy is 300 joules. What is the angular velocity? ___________

   Use the Gizmo to check your answer.

Activity C: **Continuous figures**	Get the Gizmo ready: • Click **Reset**. Turn off **Show moment of inertia**. • Set the **Piston energy** to 200 J. • In the **Objects** menu, choose **Disk**. Check that the **Radius** is 5.0 m and set the **Mass** to 1.0 kg.	

Introduction: The moment of inertia (*I*) for a single mass (*m*) that is rotating around a point with radius *r* is given by the equation $I = mr^2$. However, the equation changes when the mass is not located at a single point, but spread out over an area. In this case, some of the mass is located close to the center of rotation, where it causes less resistance to rotation.

Question: How do you calculate the moment of inertia for disks, rings, and other shapes?

1. Observe: Click **Play**. What is the angular velocity of the disk? ____________________

2. Calculate: Based on the rotational kinetic energy and angular velocity, what is the moment of inertia of this disk? ____________________ Turn on **Show moment of inertia** to check.

3. Explore: Click **Reset**.

   A. Double the mass of the disk to 2.0 kg. How does this affect the moment of inertia?

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   B. Return the mass to 1.0 kg. Record the moment of inertia for each of these radii:

   4.0 m __________ 3.0 m __________ 2.0 m __________ 1.0 m __________

   C. How is the moment of inertia related to the square of the radius? ________________

   ____

4. Analyze: The general equation for moment of inertia is $I = kmr^2$, where *k* is a constant that depends on the shape of the object.

   Based on what you have seen so far, what is the value of *k* for a disk? _________________

5. Practice: Use the Gizmo to find the value of *k* for the other objects. List their values below.

   Ring: *k* = _________________ Sprocket 1: *k* = _________________

   Sprocket 2: *k* = _________________ Sprocket 3: *k* = _________________

(Activity C continued on next page)

Activity C (continued from previous page)

6. Think and discuss: Why is the value of *k* lower for a disk than for a ring? (Hint: Remember that *k* is a constant that represents how the mass is distributed away from the center of rotation.)

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7. Explain: Why is the value of *k* the same for a ring as it is for a point mass? _______________

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8. Summarize: In general, why does the moment of inertia increase when mass is distributed farther from the center of a rotating object? Try to explain your reasoning in words rather than using equations.

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