Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Student Exploration: Polygon Angle Sum**

**Vocabulary:** diagonal, exterior angle, interior angle, polygon, regular polygon

**Prior Knowledge Questions** (Do these BEFORE using the Gizmo.)

1. **Polygons** with 3 to 8 sides have special names. Write the names in the table below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **# of sides** | 3 | 4 | 5 | 6 | 7 | 8 |
| **Name of polygon** |  |  |  |  |  |  |

1. Based on the number of sides in a pentagon, how many events do you think make up an athletic event called a pentathlon?

**Gizmo Warm-up**

In the *Polygon Angle Sum* Gizmo, you can manipulate polygons with 3 to 8 sides, and see what you can discover about their angle measures.

To change the number of sides on the polygon, drag the slider, or select the number in the text field, type in a new value, and hit **Enter**.

1. Be sure **Regular polygon** is selected. Make a variety of regular polygons by changing the number of sides and dragging the vertices. Describe how regular polygons behave.
2. Now unselect **Regular polygon**. Make a variety of irregular polygons by varying the number of sides and dragging the vertices. Describe how irregular polygons behave.
3. Based on your answers above, how do you think a **regular polygon** is defined? (Be sure to discuss side lengths and angle measures.)

|  |  |  |
| --- | --- | --- |
| **Activity A:** **Interior angle measures** | Get the Gizmo ready: * Turn on **Regular polygon**.
* Turn on **Show angle measures** and be sure **Interior angles** is selected.
 | PolygonAngleSumSE2 |

1. In the Gizmo, set **Number of sides** to 3.
	1. Make a variety of regular triangles by dragging the vertices. What is the measure of each **interior angle** of any regular triangle?
	2. What is the sum of the interior angle measures of any regular triangle?
	3. Turn off **Regular polygon**. Make a variety of irregular triangles by dragging the vertices. What is the sum of the interior angle measures of any triangle?

 Select **Show angle sum table** to check your answer.



1. Set **Number of sides** to 4 and turn on **Regular polygon**.
2. A regular quadrilateral is a square. What is the measure of each interior angle of a square?
3. What is the sum of the interior angle measures of a square?
4. Turn on **Divide into triangles**. The segment that appearsis called a **diagonal**. How many triangles does the diagonal divide the square into?
5. Turn off **Regular polygon** and reshape the quadrilateral by dragging the vertices. What is the sum of the interior angle measures of any quadrilateral?
6. Turn on **Regular polygon** and turn off **Divide into triangles**. Set **Number of sides** to 5.
7. How many triangles do you think the pentagon can be divided into?
8. Predict the sum of the measures of the interior angles of a pentagon.

Check your answers in the Gizmo.

1. Turn off **Regular polygon** and experiment with some irregular pentagons. What is the sum of the interior angle measures of any pentagon?

**(Activity A continued on next page)**

**Activity A (continued from previous page)**

1. Use what you’ve learned about triangles, quadrilaterals, and pentagons to help you fill in the following table. Fill it in on your own first. Then, check your answers in the Gizmo.

|  |  |  |  |
| --- | --- | --- | --- |
| **Polygon** | **Number of sides** | **Number of triangles** | **Sum of measures of interior angles** |
| Triangle | 3 |  |  |
| Quadrilateral | 4 |  |  |
| Pentagon | 5 |  |  |
| Hexagon |  |  |  |
| Heptagon |  |  |  |
| Octagon |  |  |  |

1. Compare the number of sides and the number of triangles for all the polygons in your table.
2. What is the relationship between the numbers in the middle two columns?

1. Based on the patterns in the table, into how many triangles can a 17-sided polygon (17-gon) be divided?
2. What is the sum of the interior angle measures of a 17-gon?
3. In general, into how many triangles can an *n*-gon be divided?
4. What is the sum of the interior angle measures of an *n*-gon?

This formula is called the *Polygon Angle Sum Theorem*.

|  |  |
| --- | --- |
| **Regular Polygon** | **Calculation of measure of each interior angle** |
| Hexagon |  |
| Octagon |  |

1. Turn on **Regular polygon**.
2. What is the measure of *each* interior angle of a regular hexagon? Of a regular octagon? Show your calculations in the table to the right.
3. In general, how can you find *each* interior angle measure for any regular polygon?

Turn on **Show angle measures** and select **Interior angles** to check your answer.

|  |  |  |
| --- | --- | --- |
| **Activity B:** **Exterior angle measures** | Get the Gizmo ready: * Be sure **Regular polygon** is selected.
 |  |

1. In the Gizmo, set **Number of sides** to 3.
	1. Sketch your triangle in the space to the right. Label each interior angle with its measure. Then extend one side. This creates an **exterior angle**.
	2. What is the relationship between the exterior angle you drew and the adjacent interior angle?
	3. What is the measure of the exterior angle? Turn on **Show angle measures** and select **Exterior angles** to check your answer.
	4. What is the sum of the exterior angle measures of a regular triangle?
2. Use the Gizmo to find the sum of the exterior angle measures for the polygons given below. Write your results in the table. Be sure to look at both regular and irregular polygons.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Polygon** | Triangle | Quadrilateral | Pentagon | Hexagon | Heptagon | Octagon |
| **Sum of ext. angle****measures** |  |  |  |  |  |  |

1. Look at the table you filled in above.
2. What do you think is the sum of exterior angle measures of any polygon?

Explain.

This is called the *Polygon Exterior Angle Sum Theorem*.

1. What do you think is the measure of *each* exterior angle of a regular 18-gon?

Explain.

1. Select **Regular polygon** and be sure **Exterior angles** is selected. Drag a vertex toward the middle of the polygon until it is almost a single point. Do this for a variety of different polygons. How does this help illustrate the *Polygon Exterior Angle Sum Theorem*?