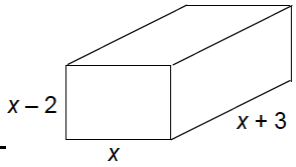
Name: _____ Date: _____

Student Exploration: Polynomials and Linear Factors

Vocabulary: degree, linear factor, multiplicity, polynomial, zero (of a polynomial)

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. A rectangular box has a width of x meters, length of $x + 3$ meters, and a height of $x – 2$ meters. Write an expression for its volume.



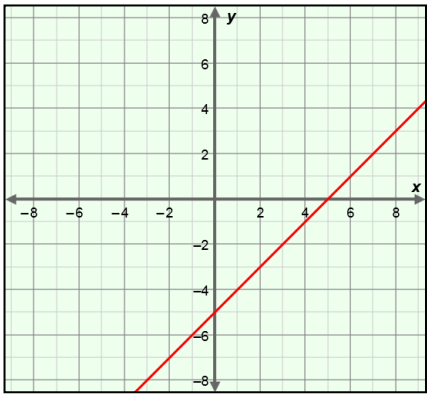
_____

2. Find the dimensions and volume of the box when $x = 6$.

   Dimensions: _____ Volume: _____

Gizmo Warm-up

A **polynomial** is a monomial or sum of monomials. In the *Polynomials and Linear Factors* Gizmo, you can create polynomials, written as products of linear factors.



1. Show $y = x – 5$ by moving the blue ***a*** slider to 5.

   A. What is the y-intercept? _____

   B. What is the value of y when $x = 6$? _____

   C. A **zero** of a polynomial is an x-value for which the polynomial is equal to zero.

   What is the zero of this polynomial? _____ How can you tell? _____

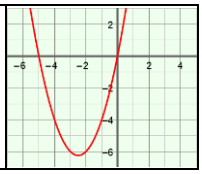
   _____

2. Select the green ***b*** slider and set ***b*** = 2 (with ***a*** = 5).

   A. Look at the polynomial in the pink box. How many linear factors does the polynomial have? _____ What are they? _____

   B. How many x-intercepts does the graph have? _____ What are they? _____

   C. What is the y-intercept? _____ Click on the **TABLE** tab to check your answers.

Activity A: **First- and second-degree polynomials**	Get the Gizmo ready: • Select the **CONTROLS** tab. • Unselect all checkboxes.	

1. Move the blue ***a*** slider to vary the value of *a* in $y = x - a$. As you do, observe the graph.

   A. What happens to the graph as you increase *a*? _____

   B. What happens to the graph as you decrease *a*? _____

   C. Drag the ***a*** slider to create the polynomial function $y = x + 5$. To create that function, what is the value of *a*? _____

   D. What is the zero of $y = x + 5$? _____

   E. What is the relationship between a linear factor and the zero of a function? _____

2. Select the ***b*** slider. Move the slider to vary *b* and observe what happens to the graph.

   A. What happens to the *x*-intercepts of the graph as you increase *b*? _____

   B. What happens as you decrease *b*? _____

   C. In general, what shape is the graph of a polynomial with two linear factors? _____

3. In the Gizmo, graph $y = (x - 3)(x - 7)$.

   A. What are the linear factors of the polynomial? _____

   B. What are the zeros of the polynomial? _____

   C. If you multiply the linear factors, what polynomial do you get? _____

   Turn on **Show unfactored form** to check your answer.

   D. What is the **degree** (greatest exponent on *x*) of the polynomial in part C? _____

   E. How does the degree relate to the number of linear factors? _____

(Activity A continued on next page)

Activity A (continued from previous page)

4. In the Gizmo, move the sliders to set ***a*** = –1 and ***b*** = –1.

   A. How many linear factors does the polynomial have? _____

      What are they? _____

   B. How many *x*-intercepts does the graph have? _____

   C. The **multiplicity** of a zero is the number of times its associated factor occurs in the polynomial. This polynomial function has a zero at $x = -1$ with multiplicity 2. Write the function in factored form below, using a squared term to represent the multiplicity.

      _____

5. The zeros of a second-degree polynomial are –4 and 7.

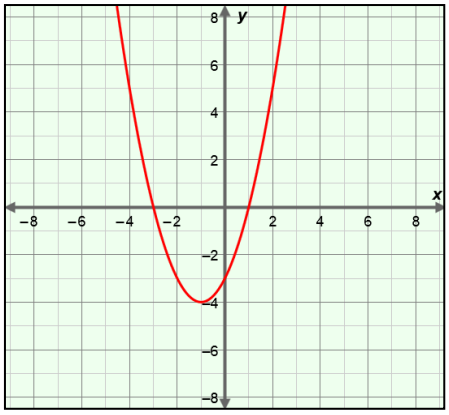
   A. What are the linear factors of the polynomial? _____

   B. Write the polynomial in factored form. _____

   C. Write the polynomial in standard form. _____

      Check your answers in the Gizmo.

6. The graph of a polynomial function is shown to the right.



   A. What degree is the polynomial? _____

      How do you know? _____

      _____

   B. What are the *x*-intercepts? _____

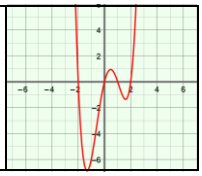
   C. What do you think its linear factors are?

      _____

   D. In factored form, write the lowest-degree polynomial function that has those linear factors.

      _____

   E. Graph your function in the Gizmo. Does your graph match the one above? _____
      If not, adjust your function until you match it.

Activity B: **Third- and fourth-degree polynomials**	Get the Gizmo ready: • Be sure the Gizmo shows the **CONTROLS** tab and that all boxes are unchecked.	

1. In the Gizmo, select the ***b*** and ***c*** sliders. Set ***a*** = 0, ***b*** = –2, and ***c*** = 3.

   A. How many linear factors does this polynomial have? _____

   B. Look at the graph. How many *x*-intercepts are there? _____

2. Consider how the values of *a*, *b*, and *c* affect the number of zeros.

   A. Vary ***a***, ***b***, and ***c*** so that the graph has exactly two distinct *x*-intercepts. What do you notice about the linear factors and zeros? _____

   _____

   B. Write the polynomial you created in factored form. _____

   C. Now rewrite it, using a squared factor. _____

   D. Can you vary ***a***, ***b***, and ***c*** so that the graph has exactly one *x*-intercept? _____

   Explain. _____

3. Graph $y = (x – 2)(x – 3)(x + 1)(x – 4)$ in the Gizmo. If you like, zoom out by clicking **–**.

   A. Describe the graph. _____

   B. How many distinct zeros does the polynomial have? _____

   C. What are the zeros? _____

   D. Can you vary ***a***, ***b***, ***c***, and ***d*** to form a polynomial with exactly 5 distinct zeros? _____

   Explain. _____

4. Write a polynomial in factored form that satisfies each set of conditions below. Check your answers in the Gizmo.

   A. Third-degree, with *x*-intercepts –2, 3, and 5 _____

   B. Third-degree, with one zero of multiplicity 3 _____

   C. Fourth-degree, with exactly two distinct roots _____