



Name: _____

Date: _____

Student Exploration: Pyramids and Cones

Vocabulary: cone, height (of a cone or pyramid), pyramid, volume

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. A cross section is the shape you'd get if you could "slice" a three-dimensional figure. Imagine taking horizontal cross sections of the glass shown here:



A. What shape would the cross sections be? _____

B. How do the cross sections change as you go from the top to the bottom of the glass?

2. Suppose a second glass has cross sections that are all the same size as the very top of the glass above. If the two glasses are the same height, which glass holds more liquid? Explain.

Gizmo Warm-up

A **pyramid** is a 3-dimensional figure with a polygonal base, and triangular lateral faces that meet at a point (the apex). A **cone** is like a pyramid with a circular base.



In the *Pyramids and Cones* Gizmo, you can explore the **volume** (total amount of space inside) of pyramids and cones. To resize a figure, drag the sliders, or click on the number in the text field next to a slider, type a new value, and hit **Enter**.

1. In the Gizmo, under **Shape of base**, select **Square**. Also be sure that **Drag to rotate** is selected. A pyramid is named by the shape of its base, so this is a square pyramid.


A. Drag the **Height** slider to the right and left. How does the pyramid change?

B. The **height** of a pyramid is really a distance. What distance is it? (Fill in the blanks.)

The height is the perpendicular distance from the _____ to the _____

2. Drag the **Base edge** slider. How does the pyramid change? _____



Activity A: Volume of pyramids	<u>Get the Gizmo ready:</u> <ul style="list-style-type: none"> • Be sure the Shape of base is set to Square. • Be sure Drag to rotate is turned on. 	
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1. In the Gizmo, set the **Base edge** of the square pyramid to 6 units, and the **Height** to 8 units.

A. What is the area of the base? _____ Turn on **Show area of base** to check.

B. Find the volume of a square prism with base edge = 6 units and height = 8 units.

_____ Select **Show prism/cylinder** to check.

C. Select **Show pyramid/cone volume**. What is the pyramid's volume? _____

D. Fill in the first row of the table for the prism and pyramid above. Then, create three more square pyramids and fill in the rest of the table, including the units. (Note: The final column is asking for the simplified ratio of the two volumes.)

Base edge (s)	Base area (B)	Height (h)	Prism volume	Pyramid volume	$\frac{\text{Pyramid volume}}{\text{Prism volume}}$
6 units		8 units			

E. In general, when a square pyramid has the same base area and height as a square prism, how are their volumes related? (Fill in the blanks in the fraction below.)

The volume of a square pyramid is $\frac{\boxed{}}{\boxed{}}$ the volume of a square prism with the same height and same base area.

Experiment with a variety of square pyramids and prisms to see if this is always true.

F. Write two formulas for the volume of a square pyramid in the blanks below. In the first formula, use B and h . In the second, use s and h .

$V =$ _____ $V =$ _____

Turn off **Show pyramid/cone volume** and **Show prism/cylinder**. Use your formulas to find the volumes of a variety of square pyramids. Then check them in the Gizmo.

(Activity A continued on next page)



Activity A (continued from previous page)

2. With **Square** selected, turn everything else off. Set **Base edge** to 5 units and **Height** to 3.5 units. The pyramid is a right pyramid, just like the other pyramids you've looked at so far.

A. In the space to the right, find the volume of this pyramid to the nearest hundredth. Show your work. Then select **Show pyramid/cone volume** to check.

B. Select **Drag to skew** and drag the apex. How does the volume of this oblique (tilted) pyramid compare to the volume of the right pyramid with the same base and height?

_____ Try more pyramids to see if this is always true.

3. Turn on **Show pyramid/cone volume** and **Show area of base**.

A. Select the base shapes shown below from the **Shape of base** menu. Then set the **Base edge** and **Height** to numbers of your choice and fill in the rest of the table.

Shape of base	Base area (B)	Height (h)	Prism volume	Pyramid volume	$\frac{\text{Pyramid volume}}{\text{Prism volume}}$
Triangle					
Pentagon					
Hexagon					

B. Do you think the formula for the volume of a square pyramid will work for pyramids with other base shapes? _____ Explain. _____

_____ Check other pyramids to verify.

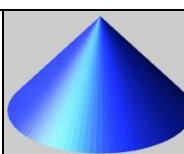
4. Select **Square** under **Shape of base**. Turn on **Show prism/cylinder**. You should see a figure like the one shown to the right (a pyramid inside a prism).



A. Suppose you had a box like this, with a pyramid inside. How much sand would this box hold, compared with an empty box of the same size?

This box would hold _____ as much as an empty box of the same size.

B. Explain why. _____

Activity B: Volume of cones	<u>Get the Gizmo ready:</u> <ul style="list-style-type: none"> • Be sure Drag to rotate is selected. • Select Circle under Shape of Base. 	
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1. For the cone shown in the Gizmo, set **Radius** to 3 units and **Height** to 7 units.

A. Use the radius to find the exact area of the base in terms of π . Show your work.

_____ Turn on **Show area of base** to check.

B. Find the volume of a cylinder with radius = 3 units and height = 7 units, in terms of π .

_____ Select **Show prism/cylinder** to check.

C. Select **Show pyramid/cone volume**. What is the cone's volume? _____

D. Fill in the first row of the table for the cylinder and cone above. Then, create two more cones and fill in the rest of the table. Use π for exact answers, and include units. (Note: The final column is asking for a ratio of the two volumes.)

Radius (r)	Base area (B)	Height (h)	Cylinder volume	Cone volume	$\frac{\text{Cone volume}}{\text{Cylinder volume}}$
3 units		7 units			

E. How are the volumes of a cone and cylinder related? _____

Experiment with a variety of cones and cylinders to see if this is always true.

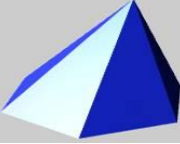
F. Write two different formulas for the volume of a cone in the blanks below. In the first formula, use B and h . In the second, use π , r , and h . Then, use your formulas to find the volumes of several cones, and check each in the Gizmo.

$V =$ _____ $V =$ _____

2. Set **Radius** to 8 units and **Height** to 6 units. Select **Drag to skew**. Drag the vertex of the cone to make it oblique. How do the volumes of the oblique and the right cones compare?

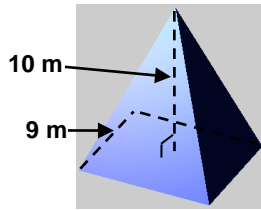
_____ Try more cones to see if this is always true.



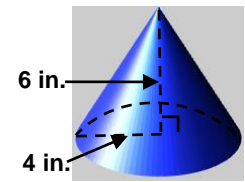
<p>Activity C: Using volume</p>	<p><u>Get the Gizmo ready:</u></p> <ul style="list-style-type: none"> • Be sure Show base and height is selected. 	
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Solve each problem. Show all of your work. Then, if possible, check your answers in the Gizmo.

1. Find the volume of the square pyramid.



4. Find the volume of the cone in terms of π .



2. An oblique triangular pyramid has a base area of 8 in.^2 and a height of 3.6 in. Find its volume.

5. An oblique cone has a base with a diameter of 12 ft and a height of 7 ft . Find its volume in terms of π .

3. A pyramid has a height of 7 cm and a volume of 21 cm^3 . What is the base area of this pyramid?

6. What is the base radius of a cone with a height of 2.5 m and a volume of $120\pi \text{ m}^3$?