Name: Date:

**Student Exploration: Quadratics in Factored Form**

**Vocabulary:** factored form of a quadratic function, linear factor, parabola, polynomial,
quadratic function, root of an equation, vertex of a parabola, *x*-intercept

**Prior Knowledge Questions** (Do these BEFORE using the Gizmo.)

1. The sides of the large rectangle to the right measure (*x* + 2) and (*x* + 1).

***x***

**1**

***x***

**2**

1. The rectangle has been divided into four regions. Label each region in the rectangle with its area.
2. What is the total area of the large rectangle?

This **polynomial** is the product of the two **linear factors**, (*x* + 2) and (*x* + 1).

1. The area of another rectangle is *x*2 + 5*x* + 6. If one side measures (*x* + 2), what is the measure of the other side?



**Gizmo Warm-up**

A function in which *y* depends on the square of *x* is a **quadratic function**. The graph of a quadratic function is a **parabola**, as shown to the right.

A quadratic function can be written in **factored form**: *y* = *a*(*x* – *r*1)(*x* – *r*2). You will explore this type of quadratic function in the *Quadratics in Factored Form* Gizmo.

To begin, set ***a*** to 1. (Change the values of ***a****,* ***r*1**, or ***r*2** by dragging the sliders, or by clicking in the text field, typing in a value, and hitting **Enter**.)

1. Turn on **Show *x*-intercepts**. Drag the ***r*1** and ***r*2** sliders to vary the values. Watch the values of the ***x*-intercepts** (the *x*-coordinates where the graph intersects the *x*-axis) as you do.

How are *r*1 and *r*2related to the *x*-intercepts?

1. Set ***a*** to 0, and then slowly drag the ***a*** slider to the right. What happens as *a* increases?

1. Set ***a*** to –1. What is true when *a* is less than zero?

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| **Activity A:** **The graph of *y* = *a*(*x* – *r*1)(*x* – *r*2)** | Get the Gizmo ready: * Turn off **Show *x*-intercepts**.
* Turn on **Show probe**.
* Set ***a*** to 1, ***r*1** to –3, and ***r*2** to 2.
 | 115SE2 |

1. The function graphed in the Gizmo should be *y* = (*x* + 3)(*x* – 2).
	1. What are the values of *r*1 and *r*2 for this equation? *r*1 = *r*2 =
	2. Drag the probe to *r*1 and then *r*2. What is the *y* value at each of these points?
	3. Evaluate *y* = (*x* + 3)(*x* – 2) at *x* = *r*1 and then at *x* = *r*2. Show your work below.
	4. Turn on **Show *x*-intercepts**. What happens when the function is evaluated at its
	*x*-intercepts?

The *x*-intercepts are the **roots**, or solutions, of the related equation (*x* + 3)(*x* – 2) = 0.

* 1. If the product of (*x* – *r*1) and (*x* – *r*2) is zero, what must be true about at least one of these factors?

This is the *zero product property*.

1. With ***a*** set to 1, vary the values of ***r*1** and ***r*2** to graph different functions of the form
*y* = (*x* – *r*1)(*x* – *r*2). What is the value of *y* = (*x* – *r*1)(*x* – *r*2) at *r*1 and *r*2?
2. Graph *y* = 4(*x* – 1)(*x* + 5) in the Gizmo.
3. What are the values of *r*1 and *r*2 for this function? *r*1= *r*2=
4. Why are *r*1 and *r*2roots of the equation 4(*x* – 1)(*x* + 5) = 0?

1. Vary ***a*** to graph different functions of the form *y* = *a*(*x* – 1)(*x* + 5). Does *a* have any effect on the roots? Explain.

**(Activity A continued on next page)**

**Activity A (continued from previous page)**

1. Set ***a*** to 1. Vary the values of ***r*1** and ***r*2** to find several parabolas with only one *x*-intercept.
2. What is the relationship between *r*1 and *r*2 when the graph has only one *x*-intercept?

1. The **vertex of a parabola** is the maximum or minimum point of the parabola.

When there is only one *x*-intercept, how are the vertex of a parabola and its
*x*-intercept related?

1. When *a* = 1, what is the factored form of a quadratic function with its vertex at the origin? Check your answer in the Gizmo.
2. While the vertex is on the *x*-axis, vary ***a***. What happens to the vertex and *x*-intercept?

Experiment with a variety of functions to check that this is always true.

1. Set ***a*** to 1. Vary the values of ***r*1** and ***r*2** to view a variety of parabolas with two
*x*-intercepts. Where is the vertex located in relationship to the two *x*-intercepts?

1. Find the quadratic function in factored form for each parabola described or shown below. Check your answers in the Gizmo by graphing your functions.
2. *x*-intercepts –4 and 0, *a* = 3

1.  *a* = 1

1. *x*-intercepts –3 and 3, *a* = –1

1.  *a* = 2

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| **Activity B:** **Factored form and polynomial form** | Get the Gizmo ready: * Be sure **Show *x*-intercepts** and **Show probe** are turned off.
* Set ***a*** to 1, ***r*1** to 3, and ***r*2** to 4.
 | 115SE5 |

1. The function graphed in the Gizmo should be *y* = (*x* – 3)(*x* – 4).
	1. You can multiply the right side of *y* = *a*(*x* – *r*1)(*x* – *r*2) to write it in **polynomial form**, *y* = *ax*2 + *bx* + *c*.

Multiply (*x* – 3)(*x* – 4) to write *y* = (*x* – 3)(*x* – 4) in polynomial form. Show your work to the right. Then select **Show polynomial form** to check your answer.

* 1. How can you combine *r*1 and *r*2 in the factored form to get *b* in the polynomial form?

* 1. How can you combine *r*1 and *r*2 to get *c*?

 Experiment with other functions to check that this is always true.

* 1. Multiply (*x* – *r*1)(*x* – *r*2) to write *y* = (*x* – *r*1)(*x* – *r*2) in polynomial form. Show your work to the right.
	2. How does the “multiplied-out” version of *y* = (*x* – *r*1)(*x* – *r*2) show how *r*1 and *r*2 can be used to find *b* and *c* in the polynomial form?

1. With ***a*** still set to 1, vary the values of ***r*1** and ***r*2** to find several parabolas with one *x*-intercept.
2. How can you use the value of *r*1 to get the value of *c* in the polynomial form?

1. How can you use the value of *r*1 to get the value of *b* in the polynomial form?

1. If *a* = 1, how can you tell if a function written in polynomial form has exactly one
*x*-intercept?

**(Activity B continued on next page)**

**Activity B (continued from previous page)**

1. Be sure **Show polynomial form** is still turned on.
2. Use the Gizmo to help you fill in the table for each of the functions in the first column.

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| --- | --- | --- | --- |
| **Factored form** | **Polynomial form when *a* = 1** | **Polynomial form when *a* = 2** | **Polynomial form when *a* = –3** |
| *y* = *a*(*x* – 2)(*x* – 4) |  |  |  |
| *y* = *a*(*x* + 1)(*x* – 2) |  |  |  |
| *y* = *a*(*x* – 5)(*x* + 2) |  |  |  |

1. How does *a* change the values of *b* and *c* in the polynomial form?

1. Use *r*1, *r*2, and *a* in the blanks below to write equations that describe the relationships you discovered above.

*b* = *c* =

1. Use the equations from above to fill the blanks below to write equations for the sum and product of the roots of a quadratic function.

*r*1 + *r*2 = *r*1*r*2 =

1. One *x*-intercept of *y* = *x*2 – 6*x* + *c* is 3.
2. How you can find the value of *c*?

1. What is the value of *c*?
2. What is true about the *x*-intercepts of this function?
3. One *x*-intercept of *y* = *x*2 + *bx* + 10 is 5.
4. How you can find the value of *b*?

1. What is the value of *b*?
2. What is the other *x*-intercept of this function?