Gizmos

Name: ______________________________ Date: ____________________

Student Exploration: Radical Functions

Vocabulary: cube root, domain, endpoint, inflection point, radical function, range, square root

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. The **square root** of a number is a value you can square to get that number. Find the square roots of the following numbers, if they exist. If there is no square root, write "none."

   $\sqrt{9}$ __________ $\sqrt{-9}$ __________ $\sqrt{49}$ __________ $\sqrt{-49}$ __________

2. The **cube root** of a number is a value you can cube to get that number. Find the cube roots of the following numbers.

   $\sqrt[3]{8}$ __________ $\sqrt[3]{-8}$ __________ $\sqrt[3]{64}$ __________ $\sqrt[3]{-64}$ __________

3. Cube roots of negative numbers exist in the set of real numbers, but square roots of negative numbers do not. Explain why this is true. ______________________________

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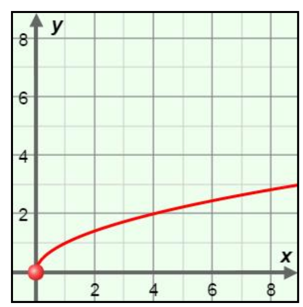
Gizmo Warm-up

Functions that include square roots or cube roots are examples of **radical functions**. In the *Radical Functions* Gizmo, you will explore the graphs and equations of square root and cube root functions.



1. In the Gizmo, be sure **Square root function** is selected. Also be sure $a = 1$, $h = 0$, and $k = 0$ so $y = \sqrt{x}$ is graphed. (To set a value, drag the slider, or type a new value in the text field and hit **Enter**.)

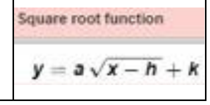
   A. Drag the ***h*** slider. How does the graph change? ______________________________

   B. Drag the ***k*** slider. How does the graph change? ______________________________

2. Select **Cube root function**. Set a to 1, h to 0, and k to 0 to graph $y = \sqrt[3]{x}$.

   A. Drag the ***h*** slider. How does the graph change? ______________________________

   B. Drag the ***k*** slider. How does the graph change? ______________________________

Activity A: Graphing square root functions	Get the Gizmo ready: • On the **CONTROLS** tab, select **Square root function**.	

1. Consider the functions $y = \sqrt{x}$ and $y = -\sqrt{x}$. (Do not graph them in the Gizmo yet.)

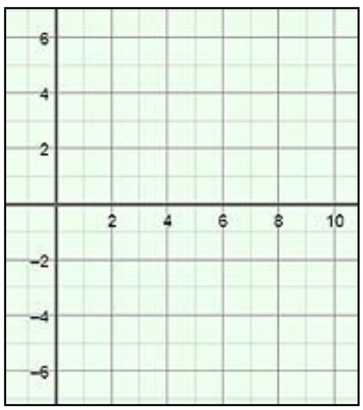
   A. How do you think their graphs differ? ______________________________

   Why? ______________________________

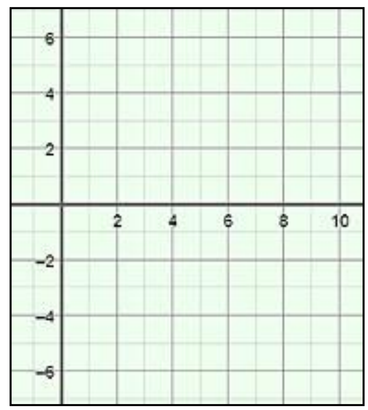
   ____

   B. Fill in the *y*-values for each of the *x*-values listed in the tables below. Use these key points to help sketch the graphs of $y = \sqrt{x}$ and $y = -\sqrt{x}$ on the coordinate planes to the right of each table. Then graph the functions in the Gizmo to check.

$y = \sqrt{x}$	
x	***y***
1	
4	
9	



$y = -\sqrt{x}$	
x	***y***
1	
4	
9	



   C. Square roots often have a "plus or minus" sign before them, to indicate two possible answers. Why is this sign not used on the *y*-values in a square root function?

   ____

   ____

2. Vary the value of ***a***.

   A. For what values of *a* is the graph increasing? ______________________________

   B. For what values of *a* is the graph decreasing? ______________________________

   C. How does the value of *a* affect the steepness of the graph? ______________________________

   ____

(Activity A continued on next page)

Activity A (continued from previous page)

3. Graph $y = \sqrt{x}$ in the Gizmo.

   A. The starting point shown on the graph is called the **endpoint**. Mouseover the endpoint. What are its coordinates? (_____, _____)

   B. Set ***h*** to 2 and ***k*** to 3. What are the coordinates of the endpoint? (_____, _____)

   C. How is the graph of $y = \sqrt{x-2} + 3$ different from the graph of $y = \sqrt{x}$? ___________

   ____

   D. Vary ***a***, ***h***, and ***k***. Does the value of *a* affect the coordinates of the endpoint? _______

   E. Experiment with other values of ***a***, ***h*** and ***k***. In general, what are the coordinates of the endpoint of the graph of $y = a\sqrt{x-h} + k$? (_____, _____)

4. The **domain** of a function is the set of all input (*x*) values of that function. The **range** is the set of all output (*y*) values.

   A. Graph $y = \sqrt{x}$ in the Gizmo. What is the domain of this function? _______________

   Turn on **Show domain** to check your answer.

   B. Explain why it makes sense that this is the domain of $y = \sqrt{x}$. __________________

   ____

   C. What is the range of $y = \sqrt{x}$? ______________ Turn on **Show range** to check.

   D. Vary ***a***, ***h***, and ***k***. In general, what are the domain and range of $y = a\sqrt{x-h} + k$?

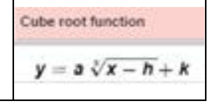
   ____

   ____

5. For each square root function below, give the coordinates of the endpoint, and the domain and range. Check your answers in the Gizmo.

   A. $y = 4\sqrt{x-2} - 5$ Endpoint: (_____, ____) Domain: ________ Range: ________

   B. $y = -3\sqrt{x+7} + 6$ Endpoint: (_____, ____) Domain: ________ Range: ________

Activity B: **Graphing cube root functions**	<u>Get the Gizmo ready</u>: • Select **Cube root function**. • Turn off **Show domain** and **Show range**.	

1. Consider the functions $y = \sqrt[3]{x}$ and $y = -\sqrt[3]{x}$. (Do not graph them in the Gizmo yet.)

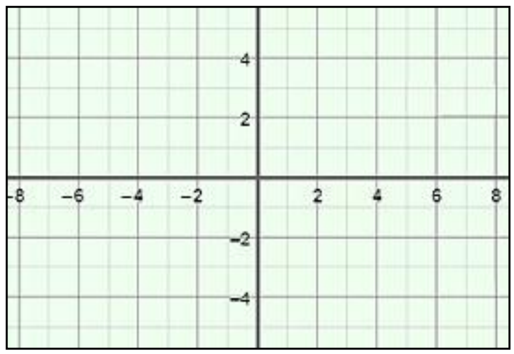
   A. How do you think their graphs differ? ___________________________________

   Why? ____

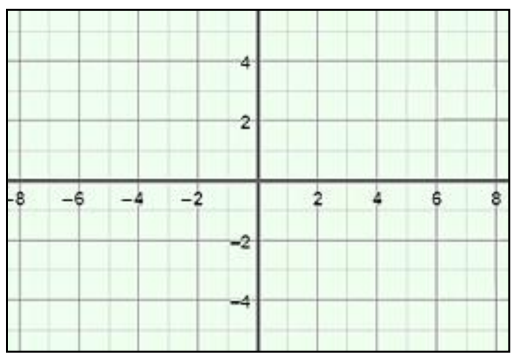
   ____

   B. Fill in the *y*-values for each of the *x*-values listed in the tables below. Use these key points to sketch the graphs of $y = \sqrt[3]{x}$ and $y = -\sqrt[3]{x}$ on the coordinate planes to the right of each table. Then check your graphs in the Gizmo.

$y = \sqrt[3]{x}$	
x	***y***
–8	
–1	
0	
1	
8	



$y = -\sqrt[3]{x}$	
x	***y***
–8	
–1	
0	
1	
8	



2. Vary the value of ***a***.

   A. For what values of *a* is the graph increasing? ___________________________

   B. For what values of *a* is the graph decreasing? ___________________________

   C. How does the value of *a* affect the steepness of the graph? ________________

   ____

(Activity B continued on next page)

Activity B (continued from previous page)

3. Graph $y = \sqrt[3]{x}$ in the Gizmo.

   A. The point shown on the graph is the **inflection point**, where the curve changes from "concave up" to "concave down."

   Mouseover the inflection point. What are its coordinates? (_____, _____)

   B. Set ***h*** to 5 and ***k*** to 2 to graph the function $y = \sqrt[3]{x-5} + 2$. What are the coordinates of the inflection point of this graph? (_____, _____)

   C. How is the graph of $y = \sqrt[3]{x-5} + 2$ different from the graph of $y = \sqrt[3]{x}$? ___________

   ____

   D. Vary ***a***, ***h***, and ***k***. Does *a* affect the coordinates of the inflection point? _____________

   E. Experiment with other values of ***a***, ***h***, and ***k***. In general, what are the coordinates of the inflection point of the graph of $y = a\sqrt[3]{x-h} + k$? (__, _____)

4. Graph $y = \sqrt[3]{x}$ in the Gizmo again.

   A. What is the domain? ________________________ Turn on **Show domain** to check.

   B. Explain why it makes sense that this is the domain. ____________________________

   ____

   C. Vary ***a***, ***h***, and ***k***. Do any of these affect the domain of the cube root function? ______

   D. What are the domain and range of all cube root functions? ______________________

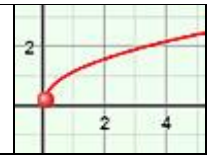
   Explain. ____

5. For each cube root function below, give the coordinates of the inflection point, and tell whether the graph is increasing or decreasing. Then check your answers in the Gizmo.

   A. $y = -\sqrt[3]{x-1} + 2$ (_____, _____) ________________

   B. $y = 2\sqrt[3]{x+7} + 5$ (_____, _____) ________________

   C. $y = -4\sqrt[3]{x+3} - 9$ (_____, _____) ________________

Activity C: **Practice with radical functions**	Get the Gizmo ready: • Select **Square root function**. • Turn off **Show domain** and **Show range**.	

1. For each square root function below, give the coordinates of the endpoint, and the domain and range. Check your answers in the Gizmo.

   A. $y = -4\sqrt{x-2} + 3$ Endpoint: (_____, _____) Domain: _________ Range: _________

   B. $y = 3\sqrt{x} - 6$ Endpoint: (_____, _____) Domain: _________ Range: _________

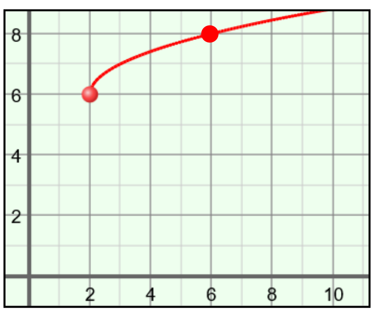
2. Write two different square root functions with a domain of $\{x \geq -3\}$, one increasing and one decreasing. Give the range of each function. Then check your answers in the Gizmo.

   Increasing: ___________________________ Decreasing: ___________________________

   Range of this function: ________________ Range of this function: ________________

3. Each graph below represents either a square root or a cube root function. Write the function below each graph. Check your answers in the Gizmo.

A.



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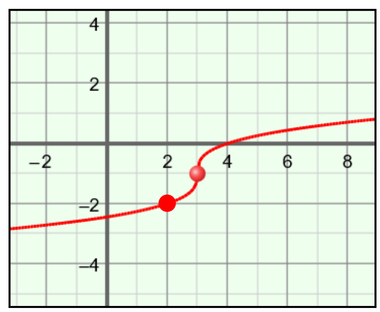
C.



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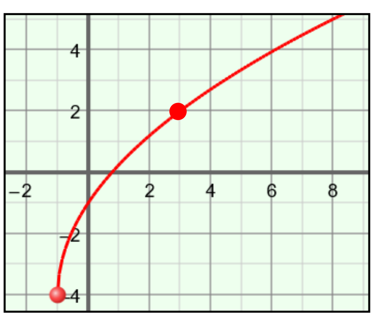
B.



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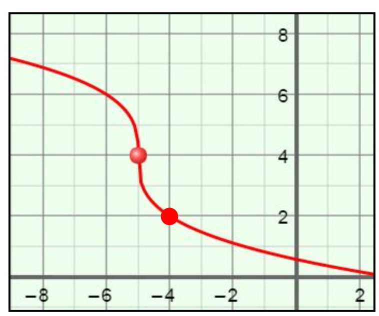
D.



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