



Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Student Exploration: Rational Functions

**Vocabulary:** asymptote, hyperbola, rational function, translation

**Prior Knowledge Questions** (Do these BEFORE using the Gizmo.)

Abby wants to enclose a rectangular area of 20 square feet to use as a garden.

1. Write a function that could be used to find the garden's length  $y$  in feet, given its width  $x$  in feet.  $y =$  \_\_\_\_\_

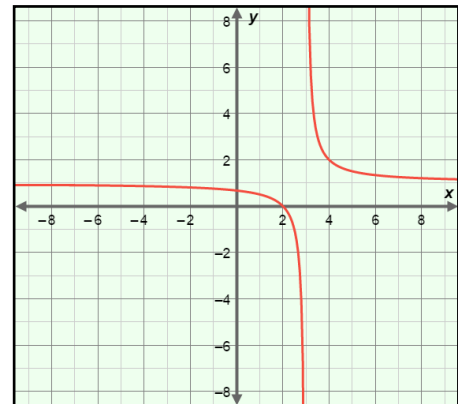


2. What happens to the length of the 20-square-foot garden as its width gets closer and closer to zero feet? \_\_\_\_\_

### Gizmo Warm-up

In the *Rational Functions* Gizmo, **rational functions** of the form  $y = \frac{a}{x-h} + k$  can be graphed. The graph of a function having this form is called a **hyperbola** and has two unconnected branches.

You can vary the values of  $a$ ,  $h$ , and  $k$  by dragging the sliders. To enter a specific value, select the number in the text field, type in the new value, and hit **Enter**.



1. Vary the values of  $h$  and  $k$ . Recall that a **translation** of a graph is a horizontal and/or vertical shift.

A. How does changing the value of  $h$  translate the graph? \_\_\_\_\_  
\_\_\_\_\_

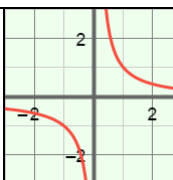
B. How does changing the value of  $k$  translate the graph? \_\_\_\_\_  
\_\_\_\_\_

2. Set  $h = 0$  and  $k = 0$ . Now vary the value of  $a$ .

A. When  $a > 0$ , in which quadrants are the branches of the graph? \_\_\_\_\_

B. When  $a < 0$ , in which quadrants are the branches of the graph? \_\_\_\_\_



|  |   |   |
|--|---|---|
| <p><b>Activity A:</b></p> <p><b>The function</b></p> $y = \frac{a}{x - h} + k$ | <p><u>Get the Gizmo ready:</u></p> <ul style="list-style-type: none"> <li>Be sure <b>Show center and asymptotes</b> and <b>Show domain</b> are turned off.</li> </ul> |  |
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- Consider the function  $y = \frac{1}{x}$ . (Do not use the Gizmo yet.)
  - What happens to the value of the function as the value of  $x$  gets very large?  
\_\_\_\_\_
  - For what value of  $x$  is the function undefined? \_\_\_\_\_ Explain why. \_\_\_\_\_  
\_\_\_\_\_
  
- In the Gizmo, graph the function  $y = \frac{1}{x}$  by setting  $a = 1$ ,  $h = 0$ , and  $k = 0$ .
  - How can you tell from the graph that the function is undefined at  $x = 0$ ?  
\_\_\_\_\_
  - Turn on **Show domain**. What value is excluded from the domain? \_\_\_\_\_
  
- With  $y = \frac{1}{x}$  graphed, select the **TABLE** tab. Set **MIN** = -0.06, **MAX** = 0.06, **STEP** = 0.01.
  - What happens to the value of the function as  $x$  approaches zero from the left?  
\_\_\_\_\_
  - What happens to the value of the function as  $x$  approaches zero from the right?  
\_\_\_\_\_
  
- An **asymptote** is a line that a graph approaches more and more closely.
  - What line appears to be the vertical asymptote of the graph? \_\_\_\_\_
  - What line appears to be the horizontal asymptote of the graph? \_\_\_\_\_

Check your answers in the Gizmo by choosing the **CONTROLS** tab and turning on **Show center and asymptotes**.

(Activity A continued on next page)



**Activity A (continued from previous page)**

5. In the Gizmo, be sure  $a = 1$  and  $k = 0$ . With **Show domain** and **Show center and asymptotes** turned on, vary the value of  $h$ .

A. How does the value of  $h$  affect the domain of the function? \_\_\_\_\_  
\_\_\_\_\_

B. Why does it make sense that the value of  $h$  affects the domain in this way? \_\_\_\_\_  
\_\_\_\_\_

C. How does the value of  $h$  affect the vertical asymptote of the graph? \_\_\_\_\_  
\_\_\_\_\_

6. Set  $a = 1$  and  $h = 0$ . Vary the value of  $k$ .

A. How does the value of  $k$  affect the horizontal asymptote of the graph? \_\_\_\_\_  
\_\_\_\_\_

B. Why does it make sense that the value of  $k$  affects the horizontal asymptote like this? \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

7. Graph  $y = \frac{2}{x}$  by setting  $a = 2$ ,  $h = 0$ , and  $k = 0$ .

A. What are the values of the function when  $x = 1$ ? \_\_\_\_\_ And  $x = 2$ ? \_\_\_\_\_

B. Now set  $a = 4$ . How does doubling the value of  $a$  affect the value of the function for a given  $x$ -value? \_\_\_\_\_

8. Predict the asymptotes and domain of each function. Check your answers in the Gizmo.

A.  $y = \frac{1}{x-3} + 4$

Asymptotes:  $x =$  \_\_\_\_\_  $y =$  \_\_\_\_\_

Domain:  
all real numbers except \_\_\_\_\_

B.  $y = \frac{5}{x+2} - 1$

Asymptotes:  $x =$  \_\_\_\_\_  $y =$  \_\_\_\_\_

Domain:  
all real numbers except \_\_\_\_\_



|  |   |                         |
|--|---|-------------------------|
| <b>Activity B:</b><br><b>Applying <math>a</math>, <math>h</math>, and <math>k</math></b> | <u>Get the Gizmo ready:</u><br><ul style="list-style-type: none"> <li>Be sure <b>Show center and asymptotes</b> and <b>Show domain</b> are turned off.</li> </ul> | $y = \frac{2}{x+3} + 1$ |
|--|---|-------------------------|

1. Consider the rational function  $y = \frac{2}{x+3} + 1$ . (Do not use the Gizmo yet.)

A. What value of  $x$  is excluded from the domain of the function? \_\_\_\_\_

Explain. \_\_\_\_\_

B. What are the equations of the vertical and horizontal asymptotes of the graph?

Vertical asymptote: \_\_\_\_\_ Horizontal asymptote: \_\_\_\_\_

C. The given function has the form  $y = \frac{a}{x-h} + k$ . What are the values of  $a$ ,  $h$ , and  $k$ ?

$a =$  \_\_\_\_\_  $h =$  \_\_\_\_\_  $k =$  \_\_\_\_\_ Check your answers in the Gizmo.

2. The graph to the right shows a rational function of the form

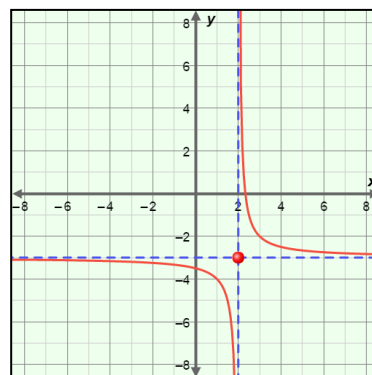
$$y = \frac{a}{x-h} + k.$$

A. Given that  $a = 1$ , what is the equation of the function?

\_\_\_\_\_

B. Explain how you know. \_\_\_\_\_

\_\_\_\_\_



\_\_\_\_\_ Then check your answer in the Gizmo.

3. A rational function of the form  $y = \frac{a}{x-h} + k$  has asymptotes at  $x = 4$  and  $y = -2$ .

A. Write the equations of *two different* functions that meet this description.

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Check your answers in the Gizmo.

B. Why is it possible to have two different rational functions with the same asymptotes?

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