Name: _____ Date: _____

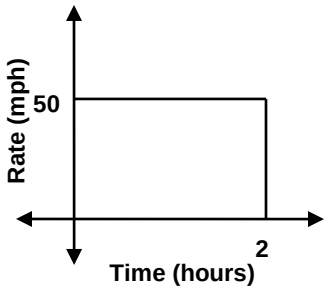
Student Exploration: Riemann Sum

Vocabulary: Riemann sum

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. Denise drove 50 miles per hour for 2 hours. How far did Denise travel? _____

2. A graph of Denise's rate vs. time is shown to the right.



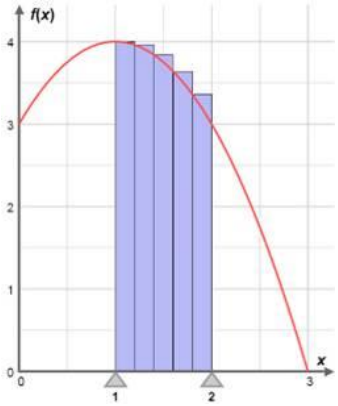
   A. What is the area of the rectangle shown in the graph?

   _____

   B. How does the area of the rectangle compare to the distance Denise traveled? _____

Gizmo Warm-up

The distance above is given by the area of the rectangle. If the area you need is under a curve, it can be tricky to calculate. You can use the areas of rectangles to estimate this area. The sum of the areas of these rectangles is called a **Riemann sum**. In the *Riemann Sum* Gizmo, you will use rectangles to estimate the area in specific intervals under a curve.



Suppose you want to find the area under $f(x) = -x^2 + 2x + 3$ from $x = 1$ to 2, as shown to the right. To show this graph in the Gizmo, select ***f*(*x*) = –*x*^{2} + 2*x* + 3** and **Show left-hand sum**, and place the arrows under the *x*-axis at 1 and 2.

1. Use the slider to set the value of ***n*** to 5, 10, and 15. (To quickly set ***n*** to a specific value, type the value in the text box to the right of the slider, and hit **Enter**.)

   A. What happens as *n* increases? _____

   B. What does *n* tell you about the graph? _____

2. The width of each rectangle is called Δ*x* ("delta *x*"), which means "change in *x*." Now vary ***n***.

   How does Δ*x* change as *n* increases? _____

Activity A: **Estimating area with rectangles**	Get the Gizmo ready: • Be sure ***f*(*x*) = –*x*^{2} + 2*x* + 3** and **Show left-hand sum** are selected, and that the arrows under the *x*-axis are at 1 and 2. • Set ***n*** to 7.	

1. You can use rectangles to estimate the area under the graph of $f(x) = -x^2 + 2x + 3$ from $x = 1$ to 2. With ***n*** set to 7, you now have a left-hand Riemann sum with 7 rectangles.

   A. Is the sum of the areas of these 7 rectangles greater than, less than, or equal to the area under the curve? _____ Why? _____

   _____

   B. Vary ***n***. Is the left-hand sum for a small *n* or large *n* closer to the actual area under the curve? _____ Why? _____

   _____

   _____

   C. Change the bounds (arrows) to $x = 0$ and 1. How is the curve from 0 to 1 different from the curve from 1 to 2? _____

   _____

   D. In general, is the left-hand sum an overestimate or underestimate of the area under each of the following types of curves?

   Increasing: _____ Decreasing: _____

   Explain why. _____

   _____

2. Select **Show right-hand sum**. (Leave **Show left-hand sum** selected too.)

   A. Vary ***n*** and watch the two sums as you do. What do you notice about the sums?

   _____

   B. What values of *n* give the best approximation of the area under a curve? _____

   _____

(Activity A continued on next page)

Activity A (continued from previous page)

3. Turn off **Show left-hand sum**. Be sure that $f(x) = -x^2 + 2x + 3$ and **Show right-hand sum** are still selected. Vary the bounds to experiment with both increasing and decreasing portions of this graph.

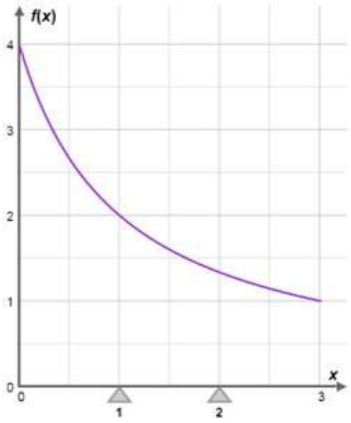
   In general, is the right-hand sum an overestimate or underestimate of the area under each of the following types of curves?

   Increasing: _____ Decreasing: _____

   Explain why. _____

   _____

4. Select $f(x) = \frac{4}{x+1}$ and set the bounds to $x = 1$ and 2. Turn off **Show right-hand sum**.



   A. Do you think a left- or right-hand sum will overestimate the area under this curve? _____

      Why? _____

      _____

      _____

      Select **Show left-hand sum** and **Show right-hand sum** to check your answer.

   B. Vary ***n***. What values of *n* give a Riemann sum that is the best approximation of the area under the curve? _____

      Explain why this makes sense. _____

      _____

      _____

   C. In theory, how many rectangles would you need in order to find the exact area under a curve? _____ Explain. _____

      _____

      _____

Activity B: **Calculating Riemann sums**	Get the Gizmo ready: • On the **CONTROLS** tab, select ***f*(*x*) = –*x*^{2} + 2*x* + 3** and **Show left-hand sum**. • Turn off **Show right-hand sum**. • Set ***n*** to 4, and the bounds (arrows) to 1 and 2.	

1. You can estimate the area under the graph of $f(x) = -x^2 + 2x + 3$ from $x = 1$ to 2 with a Riemann sum. With ***n*** set to 4, you will estimate the area with 4 rectangles.

   A. The width of each rectangle is Δx, the change in x. What is Δx? _____

   B. What five x-values evenly divide the interval from $x = 1$ to 2? 1, ____, ____, ____, 2

      These are the boundaries of the rectangles, and are called x_0, x_1, x_2, x_3, and x_4.

   C. The blue rectangles show the left-hand sum. The height of the left rectangle is $f(x_0)$.

      What notation means the heights of the other 3 blue rectangles? _____

   D. Select **Show right-hand sum**. This sum is the total area of the green rectangles.

      What notation means the heights of the 4 green rectangles? _____

   E. In the table to the right, fill in the rectangle widths under Δx. Then fill in the x_i values. Use a calculator to find the rectangle heights ($f(x_i)$) and rectangle areas ($f(x_i)\Delta x$). To check your answers, click the **TABLE** tab in the Gizmo.

i	x_i	$f(x_i)$	Δx	$f(x_i)\Delta x$
0	1			
1				
2				
3				
4	2			

   F. The left-hand sum is the sum of the areas from $i = 0$ to 3. This is given by the notation $\sum_{i=0}^{3} f(x_i)\Delta x$, where Σ is the Greek letter sigma and means "summation."

      Add the areas of the blue rectangles to find the left-hand sum. Show your work below. Then check your answers in the Gizmo.

      $\sum_{i=0}^{3} f(x_i)\Delta x$ = _____

   G. Add the areas of the green rectangles to find the right-hand sum. Show your work below. Then check your answers in the Gizmo.

      $\sum_{i=1}^{4} f(x_i)\Delta x$ = _____

(Activity B continued on next page)

Activity B (continued from previous page)

2. On the **CONTROLS** tab, select $\boldsymbol{f(x) = \frac{4}{x+1}}$. Set ***n*** to 5, and the bounds to 0 and 1.

   A. What is Δx for this Riemann sum? _____ Write this in the Δx column below.

   B. Use Δx to fill in the missing x_i values. Then use a calculator to find the values of $f(x_i)$ and $f(x_i)\Delta x$. Write all of these values in the table to the right.

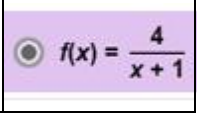
   When you are done, select the **TABLE** tab in the Gizmo to check your answers.

i	x_i	$f(x_i)$	Δx	$f(x_i)\Delta x$
0	0			
1				
2				
3				
4				
5	1			

   C. Find the left-hand sum (the sum of the blue rectangle areas) in the space below. Be sure to use a sigma (Σ) in your answer. Then check your answer in the Gizmo.

   D. Find the right-hand sum (the sum of the green rectangle areas) in the space below. Again, be sure to use a Σ in your answer. Check your answer in the Gizmo.

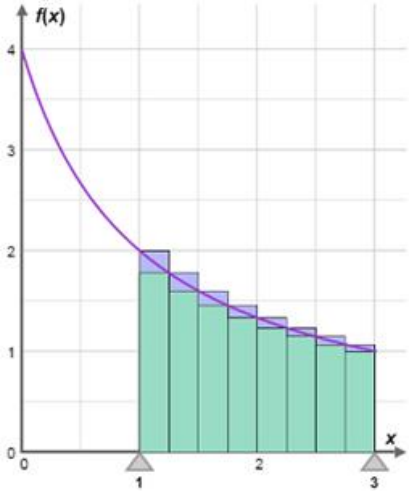
   E. Why does the right-hand sum underestimate the area under this curve? _____

Activity C: **Practice with Riemann sums**	Get the Gizmo ready: • On the **CONTROLS** tab, select $f(x) = \frac{4}{x+1}$.	

In questions 1 through 4 below, you will use a Riemann sum, with 8 rectangles ($n = 8$), to estimate the area under the graph of $f(x) = \frac{4}{x+1}$ from $x = 1$ to 3.

1. Use a calculator to fill in the values of Δx, x_i, $f(x_i)$, and finally $f(x_i)\Delta x$ in the table below.

i	x_i	$f(x_i)$	Δx	$f(x_i)\Delta x$
0				
1				
2				
3				
4				
5				
6				
7				
8				



2. With the help of the table you just made, find the left- and right-hand sums for this function. Show your work in the space below. Then check your answers in the Gizmo.

3. What must be true about the actual area under the graph of $f(x) = \frac{4}{x+1}$ from $x = 1$ to 3?

_____ Explain. _____

_____

4. What can you do to make this estimate more accurate? _____

_____