Name: _____   Date: _____

Student Exploration: Rotations, Reflections, and Translations

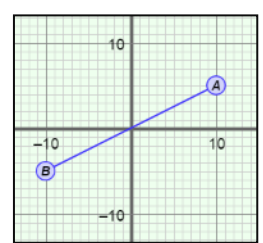**Vocabulary:** image, preimage, reflection, rotation, transformation, translation

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
Little Johnnie is playing with a puzzle piece shaped like the letter "b." He is
surprised to discover he can make other letters with the same piece.

b

1. If he flips the "b" over to the left, what new letter is formed? _____
    Draw a picture to the right.

2. If he flips the letter you drew above down, what new letter is formed? _____
    Draw a picture to the right.

3. Little Johnnie puts the "b" back in its original position. Then he rotates it 90°

    clockwise. Draw the result to the right. Is this a letter? _____

Gizmo Warm-up
The manipulations of the letter "b" are examples of different
transformations – **rotations** (turns), **reflections** (flips), and
translations (slides). In the *Rotations, Reflections, and Translations*
Gizmo™, you will rotate, reflect, and translate various figures on a
coordinate plane. To begin, select **Segment** from the **Figure type**
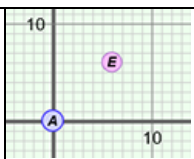menu and **Rotate around Origin** from the **Operation** menu.

1. Drag the **Rotation (in degrees)** slider. What happens to $\overline{EF}$? _____

    _____

    In all of the transformations in this Gizmo, $\overline{EF}$ is the **image** and $\overline{AB}$ is the **preimage**.

2. Select **Reflect over *x*-axis**. Drag points *A* and *B* so they are both above the *x*-axis. What do

    you notice about $\overline{EF}$? _____

3. Select **Translate**. Drag the ***x* translation** and ***y* translation** sliders. What happens to $\overline{EF}$?

    _____

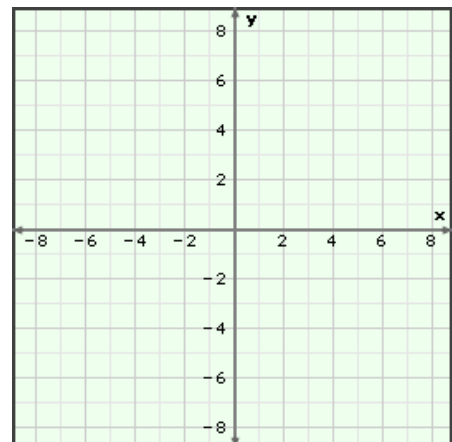| Activity A: Translations	Get the Gizmo ready: • Under **Figure type**, select **Point**. • Under **Operation**, select **Translate**.	

1. Recall that point *A* is the preimage, and point *E* is the translated image of point *A*.

   A. With the **y translation** slider set to 0, drag the **x translation** slider. How does this affect point *E*? _____

   B. Now set the **x translation** slider to 0 and drag the **y translation** slider. How does this affect point *E*? _____

   C. Set both translation sliders to a positive value. Drag point *A* around. How does this affect point *E*? _____
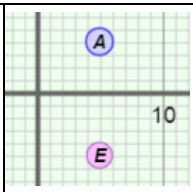
2. Turn on **Show table**. Set **x translation** to –5 and **y translation** to 6. Drag point *A* to (–2, 3).

   A. What are the coordinates of point *E*? (_____, _____)

   B. How can you calculate the coordinates of point *E*? _____

   _____

   C. Suppose a point has coordinates (*x*, *y*). What are the coordinates of the image if the *x* translation is *a* and the *y* translation is *b*? (_____, _____)

3. The endpoints of $\overline{AB}$ are at *A*(–5, 6) and *B*(4, 0). Predict the endpoints of the image $\overline{EF}$ for the translations listed in the table below. Then sketch $\overline{AB}$ and each image on the grid. Click on **Show table** to check your answers.

x and *y* translation	Point *E* Image of *A*(–5, 6)	Point *F* Image of *B*(4, 0)
x translation: 3 *y* translation: 0		
x translation: –1 *y* translation: –5		
x translation: 1 *y* translation: –6		

Activity B: Reflections	Get the Gizmo ready: • Under **Figure type**, select **Point**. • Under **Operation**, select **Reflect over *x*-axis**. • Turn off **Show table**.	

1. Recall that point *E* (the image) is the reflection of point *A* (the preimage).

   A. Drag point *A* up, down, left, and right. Fill in the table to describe how point *E* moves when you do this.

Point *A*	Up	Down	Left	Right
Point *E*				

   B. Turn on **Show table**. Watch the coordinates in the table as you drag point *A* around. How do the coordinates of point *E* compare to the coordinates of point *A*?

   _____

   _____

   C. A point has coordinates (*x*, *y*). What are the coordinates of the image if (*x*, *y*) is

   reflected over the *x*-axis? (_____, _____)

2. Turn off **Show table**. Select **Reflect over *y*-axis**.

   A. What do you think will happen to point *E* when point *A* is moved to the right?

   _____

   B. Turn on **Show table**. How do the coordinates of point *E* compare to those of point *A*?
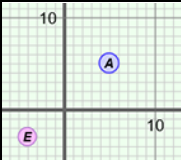
   _____

   C. A point has coordinates (*x*, *y*). What are the coordinates of the image if (*x*, *y*) is

   reflected over the *y*-axis? (_____, _____)

3. Under **Operation**, select **None**. Under **Figure type**, select **Triangle**. Drag the vertices of ΔABC to A(7, 5), B(–10, 2), and C(2, –8). Predict the coordinates of the vertices of the image ΔEFG for the reflections listed below. Then check your answers in the Gizmo.

   Over the *x*-axis: E(_____, _____)   F(_____, _____)   G(_____, _____)

   Over the *y*-axis: E(_____, _____)   F(_____, _____)   G(_____, _____)

	Get the Gizmo ready:	
Activity C: **Rotations**	• Under **Figure** type, select **Point**. • Under **Operation**, select **Rotate around Origin**. • Turn off **Show table**.	

1. Drag point *A*, the preimage, to (10, 5).

   A. Drag the **Rotation (in degrees)** slider. What shape does point *E*, the image of *A*,

      trace as you drag the slider to the right? _____

   B. Does point *E* move in a clockwise or counterclockwise direction when you drag the

      slider to the right? _____

   C. Give the angle of rotation that places point *E* at
      the coordinates listed in the table to the right.
      Click on **Show table** to check your answers.

   D. What angle of rotation brings point *E* back to

      point *A*? _____

Point *E* Image of A(10, 5)	Angle of rotation
(–5, 10)	
(–10, –5)	
(5, –10)	

2. Turn off **Show table**. Select **Segment** under **Figure type**. Set **Rotation (in degrees)** to 0°.
   Place the endpoints of $\overline{AB}$ at *A*(–1, 10) and *B*(8, –3).

   A. Predict the endpoints of the image $\overline{EF}$ for each angle of rotation listed in the table
      below. Click on **Show table** to check your answers.

Angle of rotation	0°	90°	180°	270°	360°
Point *E* Image of A(–1, 10)					
Point *F* Image of B(8, –3)					

   B. Based on the patterns you have observed, write the general coordinates of the
      image of a point with coordinates (*x*, *y*) in the table below.

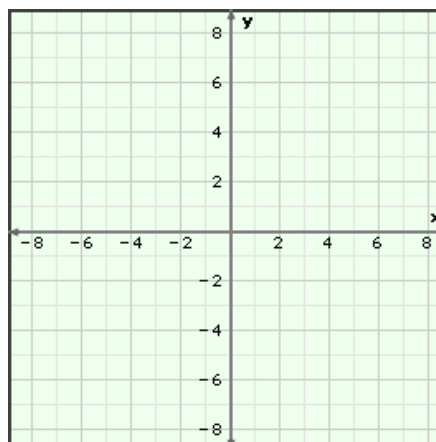| Angle of rotation	0°	90°	180°	270°	360°
Coordinates of image of (*x*, *y*)					

(Activity C continued on next page)

Activity C (continued from previous page)

3. Suppose ΔABC has the vertices listed below. Write the coordinates of the vertices of image ΔEFG for a 270° rotation in the table. Sketch both triangles on the grid to the right. Then check your answers in the Gizmo.

Vertices of ΔABC	Vertices of ΔEFG
A(2, 3)	
B(–5, 1)	
C(4, –6)	

4. <u>Extension</u>: Select **Point** under **Figure type** and **Rotate around Origin** under **Operation**. Be sure **Show table** is turned on.

    A. You can use the cosine (cos) and sine (sin) of the angle of rotation to find the image of any point on the positive x-axis. To see how this works, first use a calculator to find the cosine and sine (to the nearest hundredth) of the angles in the table below.

Angle of rotation (θ)	cos θ	sin θ	Point E Image of A(10, 0)
30°			
45°			
60°			
90°			

    B. Drag point A to (10, 0). Find the coordinates of the image point E for each angle of rotation listed above. Write the coordinates of point E in the last column of the table.

    C. Compare the cosine and sine for each angle to the coordinates of point E. What do you notice? _____

    _____

    Experiment with other points on the positive x-axis and other angles to verify.

    D. The point (x, 0) is rotated through an angle of θ when x > 0. Based on the patterns you have observed, use cosine (cos) and sine (sin) to write a formula for the coordinates of the image.

    (_____ , _____)