Name: _____ Date: _____

Student Exploration: Similarity in Right Triangles

Vocabulary: geometric mean, similar

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
Joey draws a triangle on a piece of paper. He makes an enlarged copy of his triangle.

1. **Similar** triangles are the same shape, but not necessarily the same size. Are the triangle and its copy similar? _____ Explain. _____

2. How would the side lengths of the copy compare to those of the original? _____

3. How would the angle measures of the copy compare to those of the original? _____

Gizmo Warm-up
In the *Similarity in Right Triangles* Gizmo, you can explore a right triangle and the triangles created by the altitude to its hypotenuse.

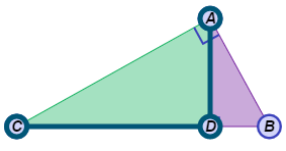
1. In the Gizmo, observe the large triangle, Δ*ABC*. Name the following parts of Δ*ABC*.

   Right angle: _____ Legs: _____ Hypotenuse: _____ Altitude: _____

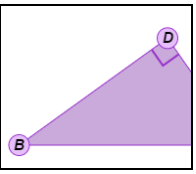
   Click on **Show side lengths** and select **Labels** to check your answers.

2. In the Gizmo, drag the vertices to form a variety of right triangles.

   A. Are both angles formed by $\overline{AD}$ and $\overline{BC}$ right angles? _____ Use the Gizmo protractor to check. (Select **Show angle measure tool** to open a Gizmo protractor. Then, attach the "donuts" to points, as shown to the right.)



   B. Are the two smaller triangles formed by $\overline{AD}$ obtuse, acute, or right? _____

Activity A: **Similar right triangles**	Get the Gizmo ready: • Turn off **Show side lengths**.	

1. In the Gizmo, click **Animate**, and then click **Flip** to get the triangles oriented the same.

   A. In the table below, list the three pairs of triangles that *appear* to be similar. Then list the three pairs of corresponding angles for each. Name each angle with three letters.

Pair of similar triangles	Corresponding pairs of angles

   B. What is true about the corresponding angles in similar triangles? _____

   _____ Drag the vertices to create a variety of triangles. Use the Gizmo protractors to check if this true for the triangles you create.

   C. You can prove that each pair of triangles is similar without measuring angles. For each pair of triangles shown below, list two pairs of corresponding angles that you know are congruent without measuring. Then state a reason for each congruent pair.

Triangles	Congruent pair of angles	Reason
Δ*ABC* and Δ*DAC*		
Δ*ABC* and Δ*DBA*		

   D. Why do you now know that Δ*ABC* ~ Δ*DAC* and Δ*ABC* ~ Δ*DBA*? _____

   _____

   E. How does that prove that Δ*DAC* ~ Δ*DBA*? _____

   _____

(Activity A continued on next page)

Activity A (continued from previous page)

2. In the Gizmo, be sure Δ1, Δ2, and Δ3 are all shown. (If you do not see all three triangles, click **Animate** and then **Flip**.) Turn off the Gizmo protractors.

   A. Name the three pairs of corresponding sides in each pair of triangles listed below.

   Δ1 and Δ2: _____ _____ _____

   Δ1 and Δ3: _____ _____ _____

   Δ2 and Δ3: _____ _____ _____

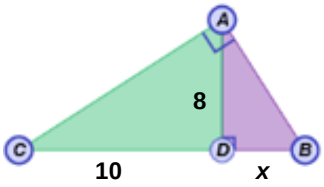
   Click on **Show side lengths** and select **Labels** to check your answers.

   B. Because the three triangles are similar, what is true about the lengths of each pair of corresponding sides? _____

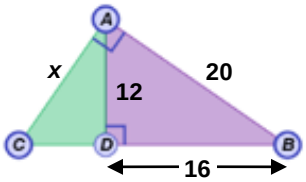
   C. Under **Show side lengths**, select **Values**. Find the ratio of each pair of corresponding side lengths. Round this ratio to the nearest hundredth. _____

3. In each triangle below, $\overline{AD}$ is the altitude to the hypotenuse of right Δ*ABC*. Use similar triangles to find *x* to the nearest tenth. Show your work. (Note: Triangles are not to scale.)

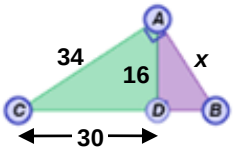
A.

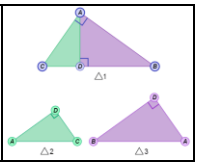


B.



C.



Activity B: **Geometric mean**	Get the Gizmo ready: • Be sure **Show side lengths** is turned off. • If Δ1, Δ2, and Δ3 are all shown, click **Animate** so only Δ1 appears.	

1. Consider the numbers 5 and 45.

   A. What number would you have to multiply by 5 to get 45? _____

   B. If you wanted to start with 5 and end up with 45 by multiplying by the same number twice, what number would you use? _____

   C. Write the sequence of three numbers you would get by doing that: 5, ____, 45

   The middle number you got above is the **geometric mean** of 5 and 45.

   D. What does the geometric mean *squared* equal? _____ What is 5 • 45? _____

   E. Write two fractions to the right:
   - 5 over the geometric mean, and
   - the geometric mean over 45.

   $\frac{\square}{\square}$ $\frac{\square}{\square}$

   F. Are the fractions equal? _____ If so, they form a proportion. In general, in a proportion of the form $\frac{a}{x} = \frac{x}{b}$, x is the geometric mean of a and b.

2. In a right triangle, the altitude to the hypotenuse divides the hypotenuse into two segments. The length of a leg is the geometric mean of the lengths of the adjacent hypotenuse segment and the whole hypotenuse.

   A. Look at the large triangle (Δ*ABC*) in the Gizmo. Write a proportion using *CD*, *AC*, and *BC* to illustrate this theorem. (*Hint:* Because the length of a leg is the geometric mean, that length appears twice in this proportion.)

   $\frac{\square}{\square} = \frac{\square}{\square}$

   B. In the Gizmo, click on **Animate** and then **Flip**. Which two similar triangles allow you to form this proportion? Δ_____ and Δ_____

   C. Which length is the geometric mean of the other two lengths? _____

   D. Use the lengths of the other leg of Δ*ABC* and its adjacent hypotenuse segment to write a proportion similar to the one you wrote above.

   $\frac{\square}{\square} = \frac{\square}{\square}$

(Activity B continued on next page)

Activity B (continued from previous page)

3. Drag the vertices of Δ1 (Δ*ABC*). Click **Show side lengths** and select **Values**.

   A. Sketch Δ*ABC* in the space to the right. Label the legs, hypotenuse, and altitude with their lengths.

   B. Use proportions to find *CD* and *BD* for the triangle you sketched above. Show your work in the space to the right.

4. In a right triangle, the length of the altitude to the hypotenuse is the geometric mean of the lengths of the segments of the hypotenuse formed by the altitude.

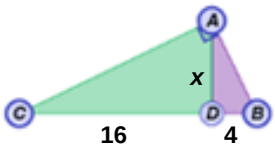
   A. In the Gizmo, drag the vertices to form a different right triangle. Under **Show side lengths**, select **Labels**. Write a proportion using *AD*, *BD*, and *CD* to illustrate this theorem. $\frac{\square}{\square} = \frac{\square}{\square}$

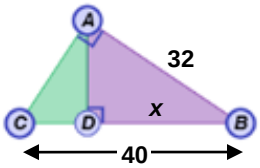
   B. Which two similar triangles allow you to form this proportion? Δ_____ and Δ_____

   C. Under **Show side lengths**, select **Values**. Sketch Δ1 in the space to the right. Label the legs, hypotenuse, and altitude with their lengths.

   D. Use proportions to find *AD* for the triangle you sketched above. Show your work in the space to the right.

5. In each triangle below, $\overline{AD}$ is the altitude to the hypotenuse of right Δ*ABC*. Use similar triangles to find *x* to the nearest tenth. Show your work. (Note: Triangles are not to scale.)

   A. 

   B. 

   C.