Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Student Exploration: Torque and Moment of Inertia**

**Vocabulary:** angular acceleration, fulcrum, lever, moment of inertia, Newton’s second law, torque, weight

**Prior Knowledge Question** (Do this BEFORE using the Gizmo.)

During recess, Tom and his little sister Marcie want to play on the see-saw. Tom is quite a bit heavier than Marcie. Where should they sit so the see-saw is balanced? Sketch their positions on the image below.

Explain your reasoning: \_\_\_\_\_\_\_\_\_\_\_\_

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**Gizmo Warm-up**

The *Torque and Moment of Inertia* Gizmo shows a see-saw, which is a type of **lever**. The see-saw can hold up to eight objects. To begin, check that the **Number of objects** is 2. Check that the mass of object **A** is 1.0 kg and the mass of object **B** is 2.0 kg. The two objects are equidistant from the triangular **fulcrum** that supports the lever.

1. Click **Release**. What happens? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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1. Click **Reset**. Without changing the masses, experiment with different positions of objects **A** and **B** by dragging them around.

Can you create a scenario in which object **A** goes down and object **B** goes up? \_\_\_\_\_\_\_

Explain: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Can you create a scenario in which object **A** perfectly balances object **B**? \_\_\_\_\_\_\_\_\_\_\_

Explain: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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| **Activity A:** **Principle of the lever** | Get the Gizmo ready: * Click **Reset**. Turn on **Show ruler**.
* Check that object **A** is 1.0 kg and **B** is 2.0 kg.
 | 522SE3 |

**Question: How can you use a light object to balance a heavy object?**

1. Explore: Experiment with the Gizmo to see how you can balance a heavy object with a light object. What do you notice about the distances of each object from the fulcrum?

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1. Gather data: For each mass and location of object **A**, find a location for object **B** so it perfectly balances object **A**. You can change the mass of object **A** by typing the mass into the text box and hitting “Enter” on your keyboard. Leave the mass of object **B** the same
(1 kg) in each experiment. Include all units in the table.

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| **Object A mass** | **Object A location** | **Object B mass** | **Object B location** | **Object A *m* × *d*** | **Object B *m* × *d*** |
| 1.0 kg | -0.4 m | 1.0 kg |  |  |  |
| 2.0 kg | -0.4 m | 1.0 kg |  |  |  |
| 3.0 kg | -0.4 m | 1.0 kg |  |  |  |
| 4.0 kg | -0.4 m | 1.0 kg |  |  |  |

1. Analyze: What patterns do you notice in your data? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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1. Calculate: Fill in the last two columns by multiplying each object’s mass by its distance from the fulcrum. The units are kg·m. (Note: The distance *d* is always a positive number.)

What do you notice? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Generalize: In general, how can you calculate the distance of object B from the fulcrum so that it balances object A? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Apply: Suppose you wanted to lift a heavy rock with a lever. Would you place the fulcrum near the rock or near the part of the lever where you are pushing? Explain.

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| **Activity B:** **Torque** | Get the Gizmo ready: * Click **Reset**. Turn on **Show initial torque**.
* Set the **Number of objects** to 1.
* Set **Mass A** to 2.0 kg.
 | 522SE4 |

**Question: What is the rotational force that an object exerts on a lever?**

1. Calculate: When object A is positioned on the see-saw, it is pulled down by the force of gravity. The gravitational force on an object, or its **weight** (*w*), is equal to its mass multiplied by gravitational acceleration (*g*). Gravitational acceleration is 9.81 m/s2 on Earth’s surface.

What is the weight of object **A**? \_\_\_\_\_\_\_\_\_\_ [Note: The unit for weight is the newton (N).]

1. Predict: The twisting force an object exerts on a see-saw is called **torque** (*τ*). How do you think the torque depends on the distance of object **A** from the fulcrum?

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1. Gather data: Place object **A** at several locations on the see-saw, on both sides of the fulcrum. Use a different mass in each experiment. In each trial, click **Release** and record the initial torque. Record object **A**’s mass, weight, location, and torque in the table below.

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| **Object A mass (kg)** | **Object A weight (N)** | **Object A location (m)** | **Object A torque (N·m)** |
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1. Analyze: Based on your data, write an equation for torque. Usethe symbol *r* to represent distance. For now, ignore the sign of the torque. Test your equation with the Gizmo.

*τ* =

1. Make a rule: Now focus on the sign of each torque value in your table. How does the sign relate to the direction of rotation? (Fill in each blank with “clockwise” or “counterclockwise.”)

*If torque is positive, the resulting motion is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.*

*If torque is negative, the resulting motion is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.*

**(Activity B continued on next page)Activity B (continued from previous page)**

1. Apply: What is the torque exerted by a 4.2-kg mass that is located 1.8 m to the right of the fulcrum? \_\_\_\_\_\_\_\_\_\_ Check your answer with the Gizmo.
2. Explore: Set the **Number of objects** to 2. Set the **Mass** of object **A** to 5 kg and its **Location** to 1.2 m. Set the **Mass** of object **B** to 3.0 kg and is **Location** to 0.5 m.
3. What torque does object **A** exert on the see-saw? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. What torque does object **B** exert on the see-saw? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
5. What do you think is the total torque on the see-saw? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
6. Check that **Show initial torque** is on and click **Release**.

What is the total torque? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Practice: A lever supports four objects:
* Object A is 3.0 kg and located 2.0 m left of the fulcrum.
* Object B is 7.0 kg and located 0.5 m left of the fulcrum.
* Object C is 8.0 kg and located 0.1 m right of the fulcrum.
* Object D is 4.5 kg and located 1.6 m right of the fulcrum.
1. What is the total torque on the lever? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

(Hint: Recall that objects to the right of the fulcrum will have a negative torque.)

Show your work:

1. When released, will the left rotate clockwise or counterclockwise? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Check your answers on the Gizmo. Were you correct? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Explain: If two kids are playing on the see-saw, why should the larger kid sit closer to the fulcrum than the smaller kid? Use the term “torque” in your explanation.

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| **Activity C:** **Moment of inertia** | Get the Gizmo ready: * Click **Reset**.
* Check that the **Number of objects** is 1.
* Set **Mass A** to 2.0 kg.
 | 522SE5 |

**Introduction:** When describing the motion of rotating objects (such as see-saws), physicists use several terms that are equivalent to those used for linear motion. For example, torque (*τ*) is the rotational equivalent of force, while **angular acceleration** (*α*) is the rotational equivalent of linear acceleration.

**Question: What factors affect how quickly a see-saw accelerates?**

1. Predict: Two children of equal mass decide to have a see-saw race. Each child sits on an identical see-saw with nothing on the other side. William sits at the end of his see-saw as far away as possible from the fulcrum. Kate sits near the middle of her see-saw close to the fulcrum. Their friends lift both see saws to the top and release them simultaneously.

 

Which see-saw do you think will hit the ground first, and why? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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1. Experiment: Move object **A** to the end of the see-saw as far as possible from the fulcrum. Click **Release**, and record how long it takes for the see-saw to hit the ground. Click **Reset** and place object **A** close to the fulcrum. Click **Release** and record the time again.

Time to hit ground when object **A** is far from the fulcrum: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Time to hit ground when object **A** is close to the fulcrum: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Compare: The rate at which a rotating object accelerates is related to its **moment of inertia**. Click **Reset**. Turn on **Show moment of inertia** and compare the moment of inertia for a mass close to the fulcrum and the same mass far from the fulcrum. (Click **Release** to see the moment of inertia.) How does moment of inertia relate to the distance to the fulcrum?

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**(Activity C continued on next page)Activity C (continued from previous page)**

1. Calculate: Place a 5.0-kg mass 2.0 m from the fulcrum. To find the moment of inertia for a mass located a distance *r* from the fulcrum, use the equation: *I* = *mr2*.

What is the moment of inertia of this mass? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (Note: Units are kg·m2.)

Check your answer by turning on **Show moment of inertia** and clicking **Release**.

1. Compare: You may have been surprised that the see-saw accelerated more slowly when the mass was far from the fulcrum and the torque was greater. That is because the angular acceleration of the see-saw depends on two factors: torque and moment of inertia (*I*). Just as mass is a measure of an object’s resistance to acceleration, moment of inertia is a measure of an object’s resistance to angular acceleration. Compare these two equations:

**Newton’s second law** (linear motion) Newton’s second law (rotational motion)

*F* = *ma τ* = *Iα*

How are these equations similar? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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1. Manipulate: Start with the rotational version of Newton’s second law (*τ* = *Iα*).
2. Solve this equation for the angular acceleration. *α* =
3. Substitute mr2 for I in this equation. *α* =
4. Recall that torque is defined as force multiplied by *α* =
radius. Substitute Fr for τ into your equation.
5. For an object on the see-saw, the force on the object is *α* =
equal to its weight, mg. Substitute mg for F in your equation.
6. Cancel everything that can be canceled to find a simplified *α* =
expression for the angular acceleration of the see-saw.
7. Interpret: Look at your expression for the angular acceleration. (If possible, check this equation with your teacher.)
8. Does the angular acceleration depend on the mass of the object? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Explain: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. How does the angular acceleration change as the distance (*r*) increases? \_\_\_\_\_\_\_\_\_ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_