Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Student Exploration:** **Vectors**

**Vocabulary:** component, dot product, magnitude, resultant, scalar, unit vector notation, vector

**Prior Knowledge Question** (Do this BEFORE using the Gizmo.)

An airplane is traveling north at 300 km/h. Suddenly, it is hit by a strong crosswind blowing 150 km/h from west to east.

Draw an arrow on the diagram showing the direction you think the plane will most likely move. Explain your answer.

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**Gizmo Warm-up**

Displacement, velocity, momentum, acceleration, and force are all examples of quantities that have both direction and **magnitude**. Anything with direction and magnitude can be represented using a **vector**.

Look at vectors **a** and **b** on the *Vectors* Gizmo grid. The initial point of each vector is shown with a circle. The terminal point of each vector is located at the tip of the arrow. Each vector is described by two **components**: the **i** component and the **j** component.

1. The two components written together make up the **unit vector notation**. What is the unit vector notation of vector **a**? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Move the initial point of vector **a** to the origin (0, 0) on the grid.
	* 1. How did the components of vector **a** change? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
		2. Drag the terminal point of vector **a** so that it lines up with the *x*-axis. Which component describes the vector’s position along the *x*-axis? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
		3. Drag the terminal point of **a** so that it lines up with the *y*-axis. Which component describes the vector’s position along the *y*-axis? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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| **Activity A:** **Vector magnitude and angle** | Get the Gizmo ready: * Change vector **a** so that its notation is 0**i** + 3**j**.
* You will need a scientific calculator for this activity.
 | 2 |

**Question: How can you determine a vector’s magnitude and angle?**

1. Observe: The magnitude of a vector is the distance from the vector’s initial point to its terminal point. The magnitude of a vector is written: ||**x**||. Magnitude is a **scalar**, or a number that does not indicate direction.
2. What is the magnitude of vector **a**? ||**a**|| = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

 Turn on **Show ruler** and use the ruler to check your answer.

1. Turn off the ruler. Drag the tip of vector **a** so that its notation is 4**i** + 3**j**. What do you think the magnitude of vector **a** is now? ||**a**|| = \_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Explore: A vector can be broken down into perpendicular vectors that describe its length along the *x* and *y* axes. Turn on **Show x, y components**. How do the **x** and **y** vectors that appearfor vector **a** relate to the **i** and **j** notation?

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1. Calculate: The *x*, *y* components of vector **a** form the two sides of a right triangle. The length of the hypotenuse of that triangle will equal the length (and, thus, the magnitude) of vector **a**.

The *Pythagorean theorem* states that for a right triangle, the square of the hypotenuse equals the sum of the squares of the other two sides:

(length of hypotenuse)2 = (length of one side)2 + (length of other side)2

Use the Pythagorean theorem to calculate the magnitude of vector **a**.

||**a**|| = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Turn on **Show ruler** and use the ruler to check your answer.

1. Apply: What are the magnitudes of the following vectors?

||3**i** – 5**j**|| = \_\_\_\_\_\_\_\_\_ ||–1**i** – 2**j**|| = \_\_\_\_\_\_\_\_\_ ||–14**i** + 3**j**|| = \_\_\_\_\_\_\_\_\_

**(Activity A continued on next page)Activity A (continued from previous page)**

1. Identify: Besides a quantity’s magnitude, vectors also indicate direction. For example, on the Gizmo’s grid, suppose the *y*-axis represents displacement to the north or south and the *x*-axis represents displacement to the east or west. Reposition vector **a** so that its notation reads 0**i** + 3**j**.

What is the direction of vector **a**: north, south, east, or west? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



1. Calculate: Move vector **a** so that its notation is 2**i** + 3**j**. Vector **a** now has a direction that is difficult to describe using words. However, the direction of vector **a** can be described as an angle (*θ*) away from the *x*-axis.

Remember that the *x*, *y* components of vector **a** form the two sides of a right triangle. For a right triangle, the tangent (tan) of any of the triangle’s angles is equal to the ratio of the opposite and adjacent sides:



From this equation, you can derive the following formula for the angle of vector **a**:



 Use a scientific calculator to find the angle of vector **a**: *θ* = \_\_\_\_\_\_\_\_\_\_\_\_

This is the angle between vector **a** and the *x*-axis (or east-west direction). Note that because the magnitudes of **x** and **y** are always positive, the angle of the vector relative to the *x* axis is positive as well.

1. Check your work: To check your calculation, select **Show angle measure tool**. Place the protractor’s center circle on the initial point of vector **a**. Place one end of the protractor on the terminal point of the **x** component and the other end on the terminal point of vector **a**.

What is the angle of vector **a**? \_\_\_\_\_\_\_\_\_\_\_\_

1. Apply: What are the angles of the following vectors?

3**i** – 5**j**: –**i** – 2**j**: –14**i** + 3**j**:

*θ* = \_\_\_\_\_\_\_\_ *θ* = \_\_\_\_\_\_\_\_ *θ* = \_\_\_\_\_\_\_\_

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| **Activity B:** **Vector Sums**  | Get the Gizmo ready: * Turn **Show x, y components** off.
* Place the initial points of vectors **a** and **b** on (0, 0).
* Set the vectors so that **a** = 5**i** + **j** and **b** = –**i** + 3**j**.
 | 4 |

**Question: How can you add vectors together?**

1. Predict: Suppose a boat is crossing a river with a swift current. In the diagram, vector **a** represents the speed and direction of the boat relative to the water, and vector **b** represents the speed and direction of the current.

On the grid at right, draw a vector to represent the resulting motion of the boat.

1. Observe: Turn on **Show resultant**. Vector **c** is the **resultant**, or the sum of vectors **a** and **b**. The resultant represents the total motion of the boat.
2. What is the angle of vector **c**? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Select **Show ruler**. What is the magnitude of vector **c**? \_\_\_\_\_\_\_\_\_\_\_
4. Analyze: Turn off the ruler. Shift vector **b** so that its initial point is on the terminal point of **a**.
5. What do you notice about the terminal point of **b**? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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1. Move **b** back to the origin, and shift **a** so that its initial point is on the terminal point of **b**. What do you notice? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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1. Infer: Now, look at the **i** and **j** components for vector **c**.
2. How is the **i** component of the resultant vector **c** related to the **i** components of vectors **a** and **b**? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. How is the **j** component of the resultant vector **c** related to the **j** components of vectors **a** and **b**? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**(Activity B continued on next page)Activity B (continued from previous page)**

1. Make a rule: How do you think the notation of **c** can be found using those of **a** and **b**?

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1. Apply: Suppose **a** = 2**i** – 3**j** and **b** = 4**i** + 0**j**.
2. Without using the Gizmo, find the resultant of adding these two vectors.

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Turn on **Show sum computation**. Were you correct? If not, what was the actual resultant? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Solve: Find the sums of the following vectors.

**a** = 5**i** – 8**j** and **b** = –4**i** – 2**j** **a** = 28**i** + 14**j** and **b** = 10**i** – 3**j**

 **c** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ **c** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**a** = 3**i** + 12**j** and **b** = –2**i** + 16**j** **a** = 5**i** – 11**j** and **b** = –6**i** – 7**j**

 **c** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ **c** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**a** = **i** – **j** and **b** = –**i** – **j** **a** = 15**i** + 10**j** and **b** = 10**i** – 20**j**

 **c** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ **c** = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Explain: Move the vectors so that **a** = –2**i** – 3**j** and **b** = 2**i** + 3**j**. Why does the resultant vector **c** no longer have an arrow? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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When two vectors cancel each other out they are said to be in a state of equilibrium.

1. Identify: Name another pair of vectors that would create a state of equilibrium.

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| **Activity C:** **Dot products**  | Get the Gizmo ready: * Turn off **Show resultant**.
* Set the vectors so that **a** = 2**i** – 3**j** and **b** = 4**i** + 5**j**.
 | 6 |

**Introduction:** While vector addition is straightforward to understand and apply, vector multiplication is not. There are several ways to express the product of two vectors, including the **dot product**.

**Question: What is a dot product?**

1. Describe: Turn on **Show dot product** and examine the calculation shown on the Gizmo.

How is a dot product found? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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1. Explore: Turn off **Show dot product**. For each combination of vectors listed in the table below, calculate the dot product. Then sketch the two vectors in the space below. Check each calculation by turning on **Show dot product**.

|  |  |  |  |
| --- | --- | --- | --- |
| **Case** | **a** | **b** | **a • b** |
| 1 | 3**i** – 2**j** | 3**i** – 2**j** |  |
| 2 | 3**i** – 2**j** | 2**i** + 3**j** |  |
| 3 | 3**i** – 2**j** | –3**i** + 2**j** |  |
| 4 | 3**i** – 2**j** | –2**i** – 3**j** |  |

|  |  |  |  |
| --- | --- | --- | --- |
| grid1 | **grid2** | **grid3** | **grid4** |

 **(Activity C continued on next page)Activity C (continued from previous page)**

1. Analyze: Look at the dot products and sketches on the previous page.
2. What is the dot product of two vectors at right angles? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. What do you notice about the dot product when the angle between the vectors is obtuse? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. Use the Gizmo to confirm these two rules. Do they hold true generally? \_\_\_\_\_\_\_\_\_\_
5. Challenge: A second way to find the dot product of two vectors is to multiply the magnitudes of the vectors, then multiply this product by the cosine (cos) of the angle (*θ*) between them:

**a** • **b** = ||**a**||·||**b**||cos(*θ*)

The dot product can be used to find the angle between two vectors. Rearrange the terms of the equation above to solve for the angle between vectors **a** and **b**.

What is the angle between **a** and **b** if **a** = 3**i** + 4**j** and **b** = 12**i** + 5**j**? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Show your work in the space below.

1. Apply: One application of the dot product is to calculate how much work is done on an object by a force. Work, a scalar quantity, is the product of force and displacement, both vector quantities (*W* = **F • d**). The unit for work is the joule (J).

Suppose vector **a** represents a force of 3**i** + 4**j** newtons that is applied to a model train on a track. Vector **b** represents the train’s displacement and is equal to 12**i** + 5**j** meters.

How much work was done on the object? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Show your work in the space below.