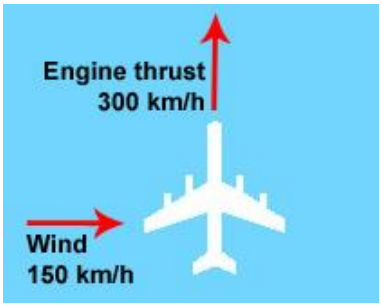
Gizmos

Name: _____ Date: _____

# Student Exploration: Vectors

**Vocabulary:** component, dot product, magnitude, resultant, scalar, unit vector notation, vector

**Prior Knowledge Question** (Do this BEFORE using the Gizmo.)




An airplane is traveling north at 300 km/h. Suddenly, it is hit by a strong crosswind blowing 150 km/h from west to east.

Draw an arrow on the diagram showing the direction you think the plane will most likely move. Explain your answer.

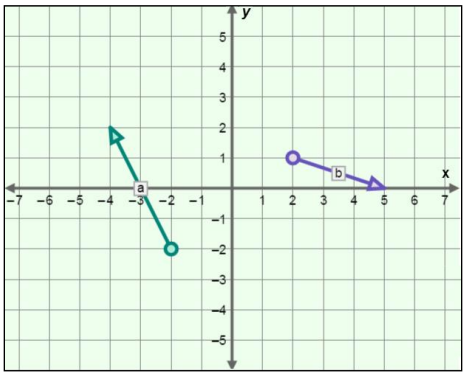
_____

_____

**Gizmo Warm-up**

Displacement, velocity, momentum, acceleration, and force are all examples of quantities that have both direction and **magnitude**. Anything with direction and magnitude can be represented using a **vector**.

Look at vectors **a** and **b** on the *Vectors* Gizmo grid. The initial point of each vector is shown with a circle. The terminal point of each vector is located at the tip of the arrow. Each vector is described by two **components**: the **i** component and the **j** component.




1. The two components written together make up the **unit vector notation**. What is the unit vector notation of vector **a**? _____

2. Move the initial point of vector **a** to the origin (0, 0) on the grid.
   A. How did the components of vector **a** change? _____
   B. Drag the terminal point of vector **a** so that it lines up with the *x*-axis. Which component describes the vector's position along the *x*-axis? _____
   C. Drag the terminal point of **a** so that it lines up with the *y*-axis. Which component describes the vector's position along the *y*-axis? _____

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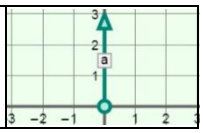
| **Activity A:**<br><br>**Vector magnitude and angle** | Get the Gizmo ready:<br>• Change vector **a** so that its notation is 0**i** + 3**j**.<br>• You will need a scientific calculator for this activity. |  |
|---|---|---|

**Question: How can you determine a vector's magnitude and angle?**

1. Observe: The magnitude of a vector is the distance from the vector's initial point to its terminal point. The magnitude of a vector is written: ||**x**||. Magnitude is a **scalar**, or a number that does not indicate direction.

   A. What is the magnitude of vector **a**? ||**a**|| = _____

   Turn on **Show ruler** and use the ruler to check your answer.

   B. Turn off the ruler. Drag the tip of vector **a** so that its notation is 4**i** + 3**j**. What do you think the magnitude of vector **a** is now? ||**a**|| = _____

2. Explore: A vector can be broken down into perpendicular vectors that describe its length along the *x* and *y* axes. Turn on **Show x, y components**. How do the **x** and **y** vectors that appear for vector **a** relate to the **i** and **j** notation?

   _____

   _____

3. Calculate: The *x*, *y* components of vector **a** form the two sides of a right triangle. The length of the hypotenuse of that triangle will equal the length (and, thus, the magnitude) of vector **a**.

   The *Pythagorean theorem* states that for a right triangle, the square of the hypotenuse equals the sum of the squares of the other two sides:

   $$(\text{length of hypotenuse})^2 = (\text{length of one side})^2 + (\text{length of other side})^2$$

   Use the Pythagorean theorem to calculate the magnitude of vector **a**.

   ||**a**|| = _____

   Turn on **Show ruler** and use the ruler to check your answer.

4. Apply: What are the magnitudes of the following vectors?

   ||3**i** – 5**j**|| = _____ ||⁻1**i** – 2**j**|| = _____ ||⁻14**i** + 3**j**|| = _____

**(Activity A continued on next page)**

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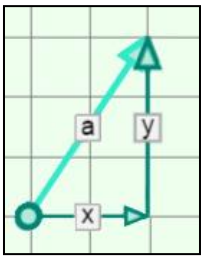
**Activity A (continued from previous page)**

5. Identify: Besides a quantity's magnitude, vectors also indicate direction. For example, on the Gizmo's grid, suppose the *y*-axis represents displacement to the north or south and the *x*-axis represents displacement to the east or west. Reposition vector **a** so that its notation reads 0**i** + 3**j**.

   What is the direction of vector **a**: north, south, east, or west? _____

6. Calculate: Move vector **a** so that its notation is 2**i** + 3**j**. Vector **a** now has a direction that is difficult to describe using words. However, the direction of vector **a** can be described as an angle ($\theta$) away from the *x*-axis.




   Remember that the *x*, *y* components of vector **a** form the two sides of a right triangle. For a right triangle, the tangent (tan) of any of the triangle's angles is equal to the ratio of the opposite and adjacent sides:

$$\tan\theta = \frac{\|\mathbf{y}\|}{\|\mathbf{x}\|}$$

   From this equation, you can derive the following formula for the angle of vector **a**:

$$\theta = \tan^{-1}\frac{\|\mathbf{y}\|}{\|\mathbf{x}\|}$$

   Use a scientific calculator to find the angle of vector **a**: $\theta$ = _____

   This is the angle between vector **a** and the *x*-axis (or east-west direction). Note that because the magnitudes of **x** and **y** are always positive, the angle of the vector relative to the *x* axis is positive as well.

7. Check your work: To check your calculation, select **Show angle measure tool**. Place the protractor's center circle on the initial point of vector **a**. Place one end of the protractor on the terminal point of the **x** component and the other end on the terminal point of vector **a**.

   What is the angle of vector **a**? _____

8. Apply: What are the angles of the following vectors?

   $3\mathbf{i} - 5\mathbf{j}$: $\theta$ = _____

   $-\mathbf{i} - 2\mathbf{j}$: $\theta$ = _____

   $-14\mathbf{i} + 3\mathbf{j}$: $\theta$ = _____

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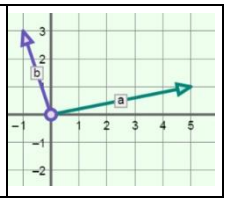
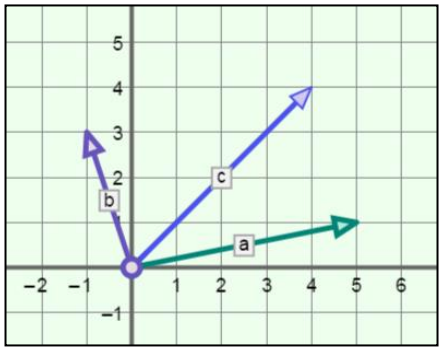
| | | |
|---|---|---|
| **Activity B:**<br>**Vector Sums** | Get the Gizmo ready:<br>• Turn **Show x, y components** off.<br>• Place the initial points of vectors **a** and **b** on (0, 0).<br>• Set the vectors so that **a** = 5**i** + **j** and **b** = ⁻**i** + 3**j**. |  |

**Question: How can you add vectors together?**

1. Predict: Suppose a boat is crossing a river with a swift current. In the diagram, vector **a** represents the speed and direction of the boat relative to the water, and vector **b** represents the speed and direction of the current.

   On the grid at right, draw a vector to represent the resulting motion of the boat.




2. Observe: Turn on **Show resultant**. Vector **c** is the **resultant**, or the sum of vectors **a** and **b**. The resultant represents the total motion of the boat.

   A. What is the angle of vector **c**? _____

   B. Select **Show ruler**. What is the magnitude of vector **c**? _____

3. Analyze: Turn off the ruler. Shift vector **b** so that its initial point is on the terminal point of **a**.

   A. What do you notice about the terminal point of **b**? _____

   _____

   B. Move **b** back to the origin, and shift **a** so that its initial point is on the terminal point of **b**. What do you notice? _____

   _____

4. Infer: Now, look at the **i** and **j** components for vector **c**.

   A. How is the **i** component of the resultant vector **c** related to the **i** components of vectors **a** and **b**? _____

   B. How is the **j** component of the resultant vector **c** related to the **j** components of vectors **a** and **b**? _____

**(Activity B continued on next page)**

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**Activity B (continued from previous page)**

5. Make a rule: How do you think the notation of **c** can be found using those of **a** and **b**?

_____

_____

6. Apply: Suppose **a** = 2**i** – 3**j** and **b** = 4**i** + 0**j**.

   A. Without using the Gizmo, find the resultant of adding these two vectors.

   _____

   B. Turn on **Show sum computation**. Were you correct? If not, what was the actual resultant? _____

7. Solve: Find the sums of the following vectors.

   **a** = 5**i** – 8**j** and **b** = ⁻4**i** – 2**j**

   **c** = _____

   **a** = 28**i** + 14**j** and **b** = 10**i** – 3**j**

   **c** = _____

   **a** = 3**i** + 12**j** and **b** = ⁻2**i** + 16**j**

   **c** = _____

   **a** = 5**i** – 11**j** and **b** = ⁻6**i** – 7**j**

   **c** = _____

   **a** = **i** – **j** and **b** = ⁻**i** – **j**

   **c** = _____

   **a** = 15**i** + 10**j** and **b** = 10**i** – 20**j**

   **c** = _____

8. Explain: Move the vectors so that **a** = ⁻2**i** – 3**j** and **b** = 2**i** + 3**j**. Why does the resultant vector **c** no longer have an arrow? _____

   _____

   When two vectors cancel each other out they are said to be in a state of equilibrium.

9. Identify: Name another pair of vectors that would create a state of equilibrium.

   _____

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| **Activity C:**<br>**Dot products** | Get the Gizmo ready:<br>• Turn off **Show resultant**.<br>• Set the vectors so that **a** = 2**i** – 3**j** and **b** = 4**i** + 5**j**. | ☑ Show dot product<br>d = a • b<br>= (5i + j) • (–2i + |
|---|---|---|

**Introduction:** While vector addition is straightforward to understand and apply, vector multiplication is not. There are several ways to express the product of two vectors, including the **dot product**.

**Question: What is a dot product?**

1. Describe: Turn on **Show dot product** and examine the calculation shown on the Gizmo.

   How is a dot product found? _____

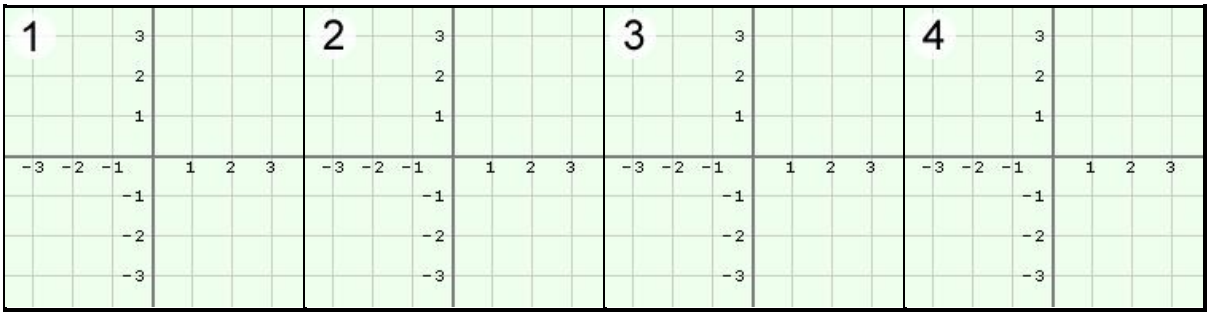
   _____

   _____

   _____

2. Explore: Turn off **Show dot product**. For each combination of vectors listed in the table below, calculate the dot product. Then sketch the two vectors in the space below. Check each calculation by turning on **Show dot product**.

| Case | a | b | a • b |
|---|---|---|---|
| 1 | 3**i** – 2**j** | 3**i** – 2**j** | |
| 2 | 3**i** – 2**j** | 2**i** + 3**j** | |
| 3 | 3**i** – 2**j** | $^{-}$3**i** + 2**j** | |
| 4 | 3**i** – 2**j** | $^{-}$2**i** – 3**j** | |




**(Activity C continued on next page)**

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**Activity C (continued from previous page)**

3. Analyze: Look at the dot products and sketches on the previous page.

   A. What is the dot product of two vectors at right angles? _____

   B. What do you notice about the dot product when the angle between the vectors is obtuse? _____

   C. Use the Gizmo to confirm these two rules. Do they hold true generally? _____

4. Challenge: A second way to find the dot product of two vectors is to multiply the magnitudes of the vectors, then multiply this product by the cosine (cos) of the angle ($\theta$) between them:

$$\mathbf{a} \cdot \mathbf{b} = ||\mathbf{a}|| \cdot ||\mathbf{b}|| \cos(\theta)$$

   The dot product can be used to find the angle between two vectors. Rearrange the terms of the equation above to solve for the angle between vectors **a** and **b**.

   What is the angle between **a** and **b** if $\mathbf{a} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = 12\mathbf{i} + 5\mathbf{j}$? _____

   Show your work in the space below.

5. Apply: One application of the dot product is to calculate how much work is done on an object by a force. Work, a scalar quantity, is the product of force and displacement, both vector quantities ($W = \mathbf{F} \cdot \mathbf{d}$). The unit for work is the joule (J).

   Suppose vector **a** represents a force of $3\mathbf{i} + 4\mathbf{j}$ newtons that is applied to a model train on a track. Vector **b** represents the train's displacement and is equal to $12\mathbf{i} + 5\mathbf{j}$ meters.

   How much work was done on the object? _____

   Show your work in the space below.

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