Name: Date:

**Student Exploration:** **Exponential Growth and Decay**

**Vocabulary:** exponential decay, exponential growth

**Prior Knowledge Questions** (Do these BEFORE using the Gizmo.)

A pair of jeans costs $40 and the sales tax is 5%.

1. How much is the sales tax? How much is the total bill?
2. What did you multiply together to find the sales tax?
3. What can you multiply by $40 to find the total bill, including tax?

Explain.

**Gizmo Warm-up**

In an exponential growth (or decay) function, as *x* increases, the *y*-values grow (or decrease) by a constant percent. In the *Exponential Growth and Decay* Gizmo, you can explore the effects of *C* and *r* in the function *y* = *C(*1 + *r*)*t*.

To vary the values of *C* and *r*, drag the sliders. To enter a specific value, click on the number in the text field, type in the new value and hit **Enter**.

1. Use the slider to vary the value of *r*.
	1. Describe the graph when *r* is greater than 0.

* 1. Describe the graph when *r* is less than 0.

1. Use the slider to vary the value of *C*. What point does *C* correspond to on the graph?

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| **Activity A:** **Exponential growth** | Get the Gizmo ready: * Select the **CONTROLS** tab.
* Unselect all checkboxes.
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1. In the Gizmo, set ***C*** to 5.00 and ***r*** to 0.20 to graph the function *y* = 5(1 + 0.2)*t*. Then select the **TABLE** tab.
2. Graph the function on the grid below. Then fill in the missing *y*-values in the table. (You can adjust the **MIN** and **MAX** values in the Gizmo table if you like.)



|  |
| --- |
| ***y* = 5(1 + 0.2)*t*** |
| ***t*** | ***y*** |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

1. What is the initial value of the function (*y*-value when *t* = 0)?

1. Each time *t* increases by 1, what happens to the *y*-values? (By what percent do they increase?)

Explain.

1. What part of the formula represents the initial value?
2. What part of the formula represents percent of growth?

The function *y* = 5(1 + 0.2)*t* is an example of an **exponential growth** function because the *y*-valuesincrease by the same percent each time *x* increases by 1.

1. Jacob currently has a summer job earning $8 per hour. Each summer he is guaranteed a 5% increase over the previous summer.
2. What is the initial value of Jacob’s hourly rate?
3. By what number is his hourly rate multiplied each year?

Explain.

1. What function models Jacob’s hourly rate, *y*, in year *t*?

Explain.

1. Use the Gizmo to check your function. Adjust your answers above if needed.

**(Activity A continued on next page)**

**Activity A (continued from previous page)**

1. The function *y* = 100(1.07)*t* represents the money in a bank account after *t* years.
2. What is the initial balance of the account?
3. What is the annual interest rate for the account?
4. Explain your answers.

1. Nina deposits $400 in an account that earns 4% interest, compounded annually.
2. What is the initial value of the account?
3. What function represents the balance after *t* years?
4. What is the value of the account after 5 years? (Use a calculator.)
5. Graph your function in the Gizmo. (Hint: Enter ***C*** = 4 in the Gizmo and let *y* = the balance of the account in hundreds.) Check your answer using the **TABLE** tab.
6. Select **Show probe**. Drag the probe slowly to the right. When will Nina have more than $600? (Click and drag the graph, or click **–** to zoom out.)
7. About how long will it take for the balance to double?
8. Suppose Nina started with $600 in the same account (with a 4% interest rate, compounded annually) instead of $400.
9. What function represents her balance after *t* years?
10. Graph your function in the Gizmo. (Use ***C*** = 6.) Drag the probe slowly to the right. About how long will it take the balance to double?
11. How long will it take to double if the initial deposit is $800?
12. What do you notice about the time it takes the balance to double?

1. What do you think would change the “doubling time”?

Explain.

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| **Activity B:****Exponential decay**  | Get the Gizmo ready:* Be sure the **CONTROLS** tab is selected and that all boxes are unchecked.
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1. In the Gizmo, set ***C*** to 5.00 and ***r*** to –0.20 to graph the function *y* = 5(1 – 0.20)*t*. Then select the **TABLE** tab.
2. Graph the function on the grid below. Then fill in the missing *y*-values in the table. (You can adjust the **MIN** and **MAX** values in the Gizmo table if you like.)



|  |
| --- |
| ***y* = 5(1 – 0.20)*t*** |
| ***t*** | ***y*** |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

1. What is the initial value of the function (*y*-value when *t* = 0)?
2. Is the function increasing or decreasing from left to right?
3. Each time *t* increases by 1, by what percent is the *y*-value changing?

Explain.

This is an example of an **exponential decay** function because the *y*-valuesare decreasing by the same percent each time *x* increases by 1.

1. Select **Show additional function**. Graph *y* = 5(1 + 0.20)*t* along with *y* = 5(1 – 0.20)*t*.
2. What do the two graphs have in common?
3. How are the graphs different?

1. A computer is purchased for $1000 and depreciates (loses value) by 15% each year.
2. What percent of value does the computer retain each year?
3. Write a function that models the value of the computer after *t* years.

Explain.

**(Activity B continued on next page)**

**Activity B (continued from previous page)**

1. Use the function *y* = 98(0.93)*t* to answer the following questions.
2. What is the *y*-intercept of the function?
3. What is the value of *r*? Explain.

1. In 2000, the population of Dullville was about 95,000. Over the next decade, the population decreased by 18%. For the questions below, assume this rate of decay will continue.
2. What function models the population after *t* decades?
3. Graph your function in the Gizmo. (Hint: Let *y* = population in thousands. To enter the initial value, type 95 in the field next to the ***C*** slider and hit **Enter**.) Then select **Show probe** and drag it to the right.

About what will the population be after 30 years (*t* = 3 decades)?

1. Roughly when will the population be *half* the initial population?
2. Use the graph to the right to answer the following questions.

1. What is the initial value?
2. What is the percent of change per year?
3. What function is graphed here?

Explain.

Check your answer in the Gizmo.

1. Challenge: Consider the graph to the right.

1. What function is graphed here?
2. Explain your reasoning.

Check your answer in the Gizmo.

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| **Activity C:** **Applications** | Get the Gizmo ready: * Select the **CONTROLS** tab.
* Select **Show probe**.
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1. Consider the function *y* = 64(1.29)*t* to answer the following questions.
2. What is the initial value? Percent of change per year?
3. Is this an exponential growth or decay function?

Explain.

1. Sandra deposits $1500 into an account that earns 2% interest compounded annually.
	1. Write a function that models the balance after *t* years.
	2. What is the balance after 7 years? (Use a calculator.)
	3. Check your answers in the Gizmo. Correct any mistakes as needed. (Hint: The Gizmo allows you to type in values for *C* from –100 to 100. For this problem, you can enter ***C*** = 15 and let *y* = the balance of the account in hundreds.)
	4. Use the probe to estimate how long it takes for the balance to double.
2. Jason buys a car for $24,000. The car depreciates (loses value) at a rate of 18% each year.
	1. What function models the value of the car after *t* years?
	2. What is the value of the car after 6 years? (Use a calculator.)
	3. Check your answers in the Gizmo. (Use ***C*** = 24.) Fix any mistakes above, if needed.
	4. About when will the car be worth *half* its initial value? (Use the probe.)
3. Use the graph to the right to answer the following questions.

1. What is the initial value?
2. What is the percent of change per year?
3. What function is graphed here?

 Explain.