Name: _____   Date: _____

Student Exploration: Exponential Growth and Decay

Vocabulary: exponential decay, exponential growth

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
A pair of jeans costs $40 and the sales tax is 5%.

1. How much is the sales tax? _____   How much is the total bill? _____

2. What did you multiply together to find the sales tax? _____

3. What can you multiply by $40 to find the total bill, including tax? _____

    Explain. _____

    _____

Gizmo Warm-up
In an exponential growth (or decay) function, as *x* increases, the *y*-values grow (or decrease) by a constant percent. In the *Exponential Growth and Decay* Gizmo, you can explore the effects of *C* and *r* in the function $y = C(1 + r)^t$.

To vary the values of *C* and *r*, drag the sliders. To enter a specific value, click on the number in the text field, type in the new value and hit **Enter**.
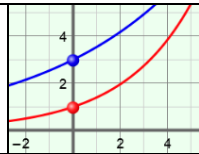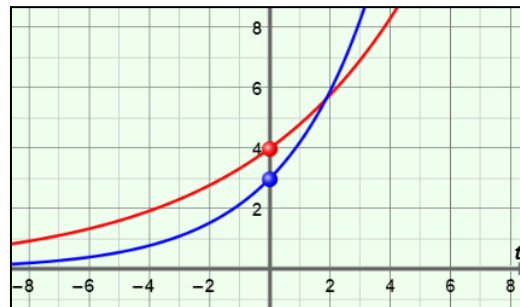

1. Use the slider to vary the value of *r*.

    A. Describe the graph when *r* is greater than 0. _____

    _____

    B. Describe the graph when *r* is less than 0. _____

    _____

2. Use the slider to vary the value of *C*. What point does *C* correspond to on the graph? _____

    _____

| **Activity A:** Exponential growth	<u>Get the Gizmo ready</u>: • Select the **CONTROLS** tab. • Unselect all checkboxes.	

1. In the Gizmo, set **C** to 5.00 and **r** to 0.20 to graph the function $y = 5(1 + 0.2)^t$. Then select the **TABLE** tab.

    A. Graph the function on the grid below. Then fill in the missing *y*-values in the table. (You can adjust the **MIN** and **MAX** values in the Gizmo table if you like.)

    B. What is the initial value of the function (*y*-value when $t = 0$)?

    _____

    C. Each time *t* increases by 1, what happens to the *y*-values? (By what percent do they increase?)

    _____

    Explain. _____

    _____
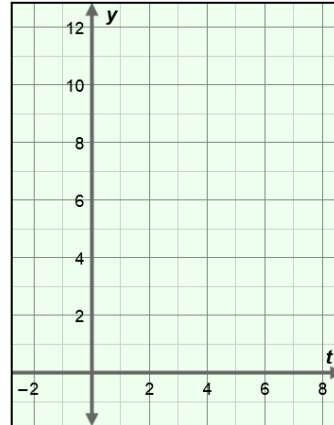
$y = 5(1 + 0.2)^t$	
t	**y**
0	
1	
2	
3	
4	

    D. What part of the formula represents the initial value? _____

    E. What part of the formula represents percent of growth? _____

    The function $y = 5(1 + 0.2)^t$ is an example of an **exponential growth** function because the *y*-values increase by the same percent each time *x* increases by 1.

2. Jacob currently has a summer job earning $8 per hour. Each summer he is guaranteed a 5% increase over the previous summer.

    A. What is the initial value of Jacob's hourly rate? _____

    B. By what number is his hourly rate multiplied each year? _____

    Explain. _____

    C. What function models Jacob's hourly rate, *y*, in year *t*? _____

    Explain. _____

    D. Use the Gizmo to check your function. Adjust your answers above if needed.

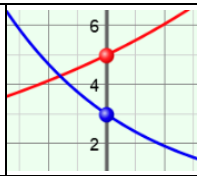**(Activity A continued on next page)**

Activity A (continued from previous page)

3. The function $y = 100(1.07)^t$ represents the money in a bank account after t years.

   A. What is the initial balance of the account? _____

   B. What is the annual interest rate for the account? _____

   C. Explain your answers. _____

   _____

4. Nina deposits $400 in an account that earns 4% interest, compounded annually.

   A. What is the initial value of the account? _____

   B. What function represents the balance after t years? _____

   C. What is the value of the account after 5 years? (Use a calculator.) _____

   D. Graph your function in the Gizmo. (Hint: Enter **C** = 4 in the Gizmo and let y = the balance of the account in hundreds.) Check your answer using the **TABLE** tab.

   E. Select **Show probe**. Drag the probe slowly to the right. When will Nina have more

   than $600? (Click and drag the graph, or click **–** to zoom out.) _____

   F. About how long will it take for the balance to double? _____

5. Suppose Nina started with $600 in the same account (with a 4% interest rate, compounded annually) instead of $400.

   A. What function represents her balance after t years? _____

   B. Graph your function in the Gizmo. (Use **C** = 6.) Drag the probe slowly to the right.

   About how long will it take the balance to double? _____

   C. How long will it take to double if the initial deposit is $800? _____

   D. What do you notice about the time it takes the balance to double? _____

   _____

   E. What do you think would change the "doubling time"? _____

   Explain. _____

   _____

| Activity B:	Get the Gizmo ready:
Exponential decay	• Be sure the **CONTROLS** tab is selected and that all boxes are unchecked.

1. In the Gizmo, set **C** to 5.00 and **r** to –0.20 to graph the function $y = 5(1 - 0.20)^t$. Then select the **TABLE** tab.

   A. Graph the function on the grid below. Then fill in the missing y-values in the table. (You can adjust the **MIN** and **MAX** values in the Gizmo table if you like.)

   B. What is the initial value of the function (y-value when $t = 0$)? _____

   C. Is the function increasing or decreasing from left to right? _____

   D. Each time t increases by 1, by what percent is the y-value changing?

   _____
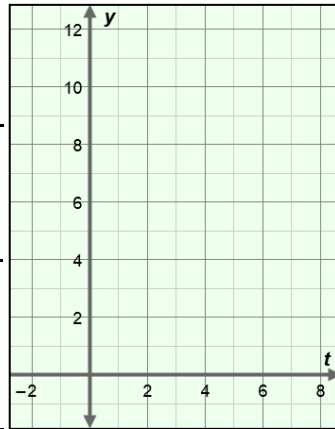
$y = 5(1 - 0.20)^t$	
t	**y**
0	
1	
2	
3	
4	

   Explain. _____

   _____

   This is an example of an **exponential decay** function because the y-values are decreasing by the same percent each time x increases by 1.

2. Select **Show additional function**. Graph $y = 5(1 + 0.20)^t$ along with $y = 5(1 - 0.20)^t$.

   A. What do the two graphs have in common? _____

   B. How are the graphs different? _____

   _____

3. A computer is purchased for $1000 and depreciates (loses value) by 15% each year.

   A. What percent of value does the computer retain each year? _____

   B. Write a function that models the value of the computer after t years. _____

   Explain. _____

(Activity B continued on next page)

Activity B (continued from previous page)

4. Use the function $y = 98(0.93)^t$ to answer the following questions.

   A. What is the *y*-intercept of the function? _____

   B. What is the value of *r*? _____   Explain. _____

   _____

5. In 2000, the population of Dullville was about 95,000. Over the next decade, the population decreased by 18%. For the questions below, assume this rate of decay will continue.

   A. What function models the population after *t* decades? _____

   B. Graph your function in the Gizmo. (Hint: Let *y* = population in thousands. To enter the initial value, type 95 in the field next to the **C** slider and hit **Enter**.) Then select **Show probe** and drag it to the right.

      About what will the population be after 30 years (*t* = 3 decades)? _____

   C. Roughly when will the population be *half* the initial population? _____

6. Use the graph to the right to answer the following questions.

   A. What is the initial value? _____

   B. What is the percent of change per year? _____

   C. What function is graphed here? _____

      Explain. _____
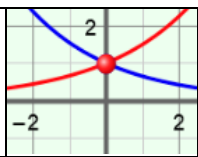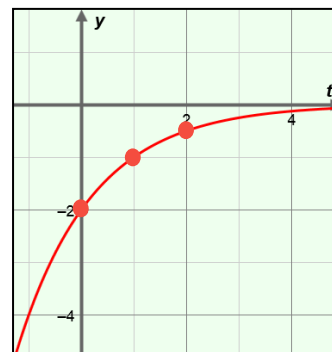
      _____

   Check your answer in the Gizmo.

7. <u>Challenge</u>: Consider the graph to the right.

   A. What function is graphed here? _____

   B. Explain your reasoning. _____

   _____

   _____

   Check your answer in the Gizmo.

Activity C: **Applications**	<u>Get the Gizmo ready</u>: • Select the **CONTROLS** tab. • Select **Show probe**.	

1. Consider the function $y = 64(1.29)^t$ to answer the following questions.

    A. What is the initial value? _____ Percent of change per year? _____

    B. Is this an exponential growth or decay function? _____

    Explain. _____

2. Sandra deposits $1500 into an account that earns 2% interest compounded annually.

    A. Write a function that models the balance after t years. _____

    B. What is the balance after 7 years? (Use a calculator.) _____

    C. Check your answers in the Gizmo. Correct any mistakes as needed. (Hint: The
       Gizmo allows you to type in values for C from –100 to 100. For this problem, you can
       enter $C = 15$ and let y = the balance of the account in hundreds.)

    D. Use the probe to estimate how long it takes for the balance to double. _____

3. Jason buys a car for $24,000. The car depreciates (loses value) at a rate of 18% each year.

    A. What function models the value of the car after t years? _____

    B. What is the value of the car after 6 years? (Use a calculator.) _____

    C. Check your answers in the Gizmo. (Use $C = 24$.) Fix any mistakes above, if needed.

    D. About when will the car be worth *half* its initial value? (Use the probe.) _____

4. Use the graph to the right to answer the following questions.

    A. What is the initial value? _____

    B. What is the percent of change per year? _____

    C. What function is graphed here? _____

    Explain. _____

    _____