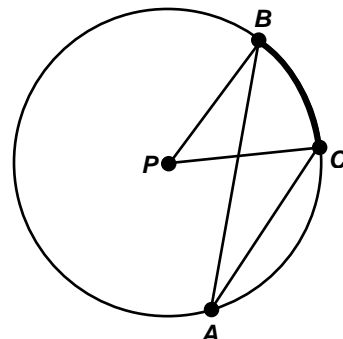Name: _____   Date: _____

Student Exploration: Inscribed Angles

Vocabulary: arc, central angle, chord, circle, inscribed angle

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. In **circle** *P* shown to the right, the two segments with both endpoints on the circle are **chords** ($\overline{BA}$ and $\overline{CA}$).

    A. Which angle has a vertex at the center of the circle?

        _____ This is a **central angle** of the circle.

    B. Which angle has a vertex on the circle and sides that are chords? _____

        This is an **inscribed angle** of the circle.

2. Both of the angles shown above intercept the same **arc** ($\overset{\frown}{BC}$). Are the measures of the

    angles the same? _____ If not, which angle is larger? _____

Gizmo Warm-up
In the *Inscribed Angles* Gizmo, you will explore the relationships between inscribed angles and the arcs they intercept. To begin, be sure **Inscribed angle 1** is selected.
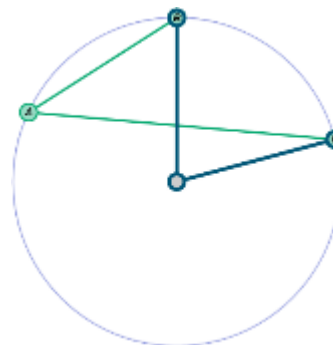
1. Select **Show angle measure tool** to open a Gizmo protractor. Attach the middle "donut" of the protractor to the center of the circle, and the other two "donuts" to points *B* and *C*. Notice that protractor forms a central angle.

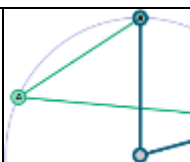   What is the measure of the central angle? _____

2. Select **Show arc measure tool** and attach the "donuts" to the endpoints of $\overset{\frown}{BC}$.

    A. What is the degree measure of $\overset{\frown}{BC}$? _____

    B. How do the degree measures of $\overset{\frown}{BC}$ and the central angle compare? _____

        _____ Drag points *B* and *C* to see if this is always true.

Activity A: **Inscribed angles and arcs**	<u>Get the Gizmo ready</u>: • Be sure **Inscribed angle 1** is still selected. • Check that the Gizmo protractor is still selected. • Turn off the Gizmo arc measure ruler.

1. The protractor should be attached to the center of the circle and to points *B* and *C*.

   A. Pull the vertex of the protractor off the center of the circle, and slowly drag it away

   from the arc. What happens to the measure of the angle? _____

   B. Drag the vertex of the protractor to point *A*. Drag point *A* around without changing *B*

   or *C*. What do you notice about the measure of ∠*BAC*? _____

   C. Use the Gizmo arc measure ruler to measure $\overset{\frown}{BC}$, the arc intercepted by ∠*BAC*.

   How are the measures of ∠*BAC* and $\overset{\frown}{BC}$ related? _____

2. With the Gizmo protractor and arc measure ruler in place, drag points *A*, *B*, and *C* to form a variety of inscribed angles and arcs. For two of these angles, record the degree measures.

   m∠*BAC* = _____   *m*$\overset{\frown}{BC}$ = _____   *m*∠*BAC* = _____   *m*$\overset{\frown}{BC}$ = _____

   A. In general, how does the measure of an inscribed angle compare to the measure of

   the arc it intercepts? _____

   B. How do the measures of a central angle and an inscribed angle that intercept the

   same arc compare? _____

3. Turn off the arc measure rulers and protractors. With **Inscribed angle 1** selected, turn on **Show diameter**. Drag points *B* and *C* so they are at the endpoints of the diameter. Notice that the diameter is a central angle with a measure of 180°.

   A. What are the measures of the arcs (semicircles) formed by the diameter? _____

   Use the Gizmo arc measure ruler to check.

   B. What is *m*∠*BAC*? _____ Use the Gizmo protractor to check.

   C. Drag point *A* around to view a variety of angles inscribed in a semicircle. What is true

   about the measure of any angle inscribed in a semicircle? _____

(Activity A continued on next page)

Activity A (continued from previous page)

4. Turn off the arc measure ruler, the protractor, and **Show diameter**. With **Inscribed angle 1** selected, turn on **Inscribed angle 2**.

    A. Drag point *F* on top of point *B* and point *E* on top of point *C*. Place point *D* close to point *A*, but not right on it.

    Do you think ∠*BAC* is congruent to ∠*FDE*? _____ Explain. _____

    _____

    Use the Gizmo protractors to check.

    B. With the Gizmo protractors in place, drag the points on the circle to see if this is true for all pairs of inscribed angles that intercept the same arc. In general, what is true about the measures of two inscribed angles that intercept the same arc?

    _____

5. Find the missing measurement(s) for each circle. Show your work. Check your answers in the Gizmo.

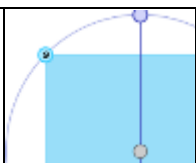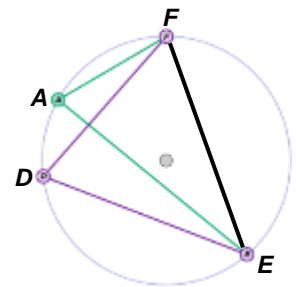   A. If $m\overset{\frown}{BC} = 210°$, find $m\angle BAC$.
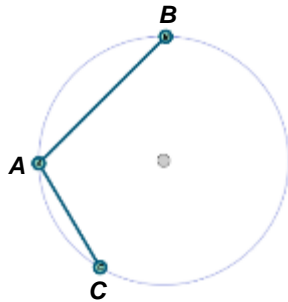
    B. If $m\angle FAE = 54°$, find $m\angle FDE$ $m\angle FPE$, and $m\overset{\frown}{FE}$.
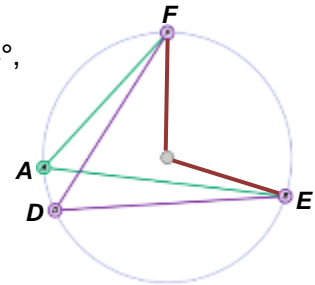
6. <u>Challenge:</u> In the figure to the right, $\overset{\frown}{FE}$ is intercepted by ∠*FAE* and ∠*FDE*.

    Is Δ*FAE* similar to Δ*FDE*? _____ Explain. _____

    _____

    _____

Activity B: **Inscribed quadrilaterals**	<u>Get the Gizmo ready</u>: • Turn off **Inscribed angle 1**, **Inscribed angle 2**, and the Gizmo measuring tools. • Select **Inscribed quadrilateral**.	

1. Drag the points on the circle to form a quadrilateral that is not a rectangle.

    A. Use the Gizmo protractors to measure opposite angles ∠G and ∠I. Then find the sum of the angle measures.

    $m\angle G =$ _____ $m\angle I =$ _____ $m\angle G + m\angle I =$ _____

    B. With the protractors attached, drag the points on the circle to create other quadrilaterals. Find the sum of the angle measures for each quadrilateral. Fill in the table to the right for three of the quadrilaterals you create.

m∠G	*m∠I*	*m∠G + m∠I*

    C. What is the sum of ∠G and ∠I in each inscribed quadrilateral? _____

    D. The sum of the angle measures of a quadrilateral is always 360°. Use this to find the

    sum of the measures of the other pair of opposite angles, ∠H and ∠J. _____
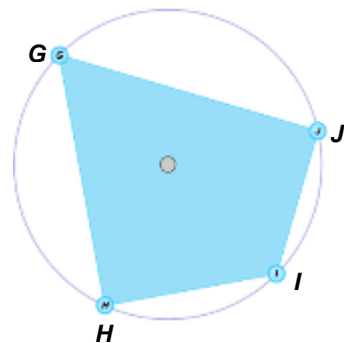
    Check your answer using the Gizmo.

    E. In general, what is the relationship between the opposite angles of a quadrilateral

    inscribed in a circle? _____

    F. Use what you know about inscribed angles to explain why this relationship is true.

    _____

    _____

    _____

2. In the figure to the right, $m\angle G = 68°$ and $m\angle H = 90°$.

    A. Find $m\angle J$ and $m\angle I$. Show your work. Then check your answers in the Gizmo.

    B. What do you know about $\overline{GI}$? _____