

Date:

Student Exploration: Number Systems

Vocabulary: base-10, binary system, digit, place value

Prior Knowledge Questions (Do these BEFORE using the Gizmo.) An odometer, like the one shown to the right, shows a car's mileage. When a car is new, every digit of the odometer is a zero: 000,000 miles. (Each individual numeral, 0 through 9, that makes up a number is called a **digit**.)

- 1. Most odometers display 6 digits. Suppose you notice that every digit on your odometer is a zero, except for one of them, which is a 1. What could the mileage of your car be? Explain.
- 2. Suppose the six digits on your odometer are three 0's, a 2, a 3, and an 8. If the mileage is

less than 1000, what is the largest it could be? _____ The smallest? _____

Gizmo Warm-up



Numbers, like mileage on an odometer, can have the same digits but very different values. This is because the value of a digit depends on its **place value**. In a **base-10 system** like we use, place values are powers of 10 such as ones (10⁰), tens (10¹), hundreds (10²), etc. In the *Number Systems* Gizmo, you can model numbers with beads to help explore place value. You can also experiment with other number systems, like binary (base-2).

Drag the **Base** and the **Base-10 value** sliders to vary the values. You can also drag one bead at a time into a column. To enter a specific value, click in the text field, type the value, and hit **Enter**.

- 1. In the Gizmo, set the Base-10 value to 238. Be sure the Base is 10.
 - A. How many beads are in each column? 10^3 : _____ 10^2 : _____ 10^1 : _____ 10^0 : _____

B. Which column corresponds to the ones digit?

2. Click Clear. What is the smallest number you can model with one bead?

What is the largest number you can model with just one bead?

Activity A:	Get the Gizmo ready:		8	
Exploring base-10	 Click the Clear button. Be sure that the Base is set to 10. 	0		

1. In the Gizmo, drag a bead from the bin and place it in the 10^2 column.

- A. When one bead is in the 10² column what is the value?
- B. Move the bead one column to the right (to 10¹). What is the new value?
- C. Move the bead one column to the right (to 10⁰). What is the new value?
- D. As the bead moves one place value to the right, how does the value change?
- E. What is the value of 10°?
- 2. Without the Gizmo, consider the base-10 number 427.
 - A. If you modeled 427, how many beads would be in each column?

10²: _____ 10¹: _____ 10⁰: _____

- B. In the Gizmo, set the **Base-10 value** to 427. Click **Show expanded form**. Using exponents, express the number 427 in expanded form.
- C. In the number 427, 4 is in the "hundreds place," 2 is in the "tens place," and 7 is in the "ones place." Look at the expanded form and explain why that makes sense.
- D. Drag a new bead from the bin on the right to the tens place (10¹ column). How does

this change the number modeled?

E. What number will be modeled if a new bead is placed in the 10⁴ column? Explain your answer. Then try it in the Gizmo.

(Activity A continued on next page)

Activity A (continued from previous page)

3.	Click Clear.	Then model the number 350 by dragging beads into the column	۱S.
<u>.</u>		Then meddel and manipel eee by anagging beade into the obtaining	

A.	How many	/ beads are in	each column? 10^3 :	10 ² :	10 ¹ :	10 ⁰ :	

- B. Express 350 in expanded form.
- C. How many beads did you use, total?
- D. What is the sum of the digits in the number 350?
- 4. Click **Clear**. Using the beads, model the number 23,062.
 - A. How many beads did you place in each column?

$$10^{5}$$
: _____ 10^{4} : _____ 10^{3} : _____ 10^{2} : _____ 10^{1} : _____ 10^{0} : _____

B. On the line below, express 23,062 in expanded form.

Check your answer in the Gizmo.

- 5. The beads in the chart on the right model a number in base-10.
 - A. How many beads are in each column?
 - 10³: _____ 10²: _____ 10¹: _____ 10⁰: _____
 - B. What number is modeled?
 - C. Express the number in expanded form.

Check your answer in the Gizmo.

- 6. In the Gizmo, click **Clear**. Then experiment with modeling numbers using 10 beads total.
 - A. What is the largest number that can be modeled using 10 beads?

Explain.

B. What is the smallest number that can be modeled using 10 beads?

Explain.





Activity B:	Get the Gizmo ready:	2 ³	2 ²	2 ¹	2 ⁰
Exploring the binary system	 Click the Clear button. Select Show expanded form. 				

The **binary system**, a base-2 number system, is widely used by computers. The only digits in a binary number are 0 and 1. A number in binary is written with a subscript of 2, like this: 101_2 .

- 1. In the Gizmo, set the **Base** to 2. Drag one bead to the rightmost column.
 - A. What base-2 (binary) number is modeled?
 - B. What is the base-10 equivalent of this binary number? _____
 - C. Now add a new bead, in the 2¹ column. What binary number is modeled?
 - D. Write the number in expanded form.
 - E. What is the base-10 equivalent of this new binary number?

Check your answers in the Gizmo.

2. A binary number is modeled to the right.
A. What binary number is modeled?
B. Write the number in expanded form.

C. What is the equivalent base-10 value? _____ Check your answers in the Gizmo.

D. What are the place values of a five-digit binary number? Explain your answer.

3. In the Gizmo, use beads to model the binary numbers 111_2 and 1000_2 .

- A. What is the base-10 value of 111₂? _____ 1000₂? _____
- B. You should have found that the values of 111_2 and 1000_2 are only one apart. Explain why this makes sense.

22



2⁰

21

2⁰

