Name: ______________________________________ Date: ________________________

Student Exploration: Permutations and Combinations

Vocabulary: combination, factorial, permutation

Prior Knowledge Question (Do this BEFORE using the Gizmo.)

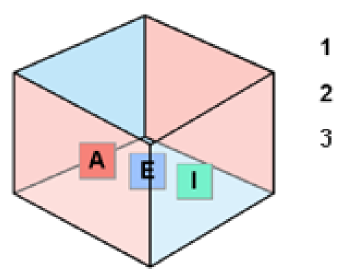
1. Suppose you have a quarter, a dime, and a penny in your pocket. If you grab the coins one at time, what are the different orders you can pull them out of your pocket? (Use Q for quarter, D for dime, and P for penny.)

   ____

2. Suppose you are buying a stick of gum that costs 36 cents. Does it matter in what order you pull out the coins? _________ Explain why or why not. ______________________________

   ____

Gizmo Warm-up
The different ways of ordering coins are examples of **permutations**. Permutations are arrangements of objects in which order matters, such as QDP and DPQ. A **combination** is a collection of objects in which the order does not matter. (QDP and DPQ – and other arrangements of the same 3 letters – are considered the same combination.) You can study these topics in the *Permutations and Combinations* Gizmo.



1. Check that the **Number of tiles in box** is 3. (The tiles have the letters A, E, and I.) Set the **Number of draws from box** to 3, and check that **Is order important?** is set to **Yes**.

   A. Click **Simulate** 8 times. Record your results below.

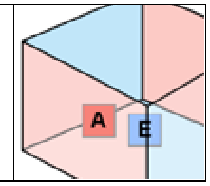
   ______ ______ ______ ______ ______ ______ ______ ______

   B. How many different permutations of 3 letters did you get? _________

2. Look at the **TREE** tab on the right side of the Gizmo.

   A. How many permutations are there of A, E, and I? _________

   B. What are they? ____

Activity A: **Permutations**	Get the Gizmo ready: • Set the **Number of tiles in box** to 2. • Set the **Number of draws from box** to 1. • Next to **Is order important?** select **Yes**.	

1. Suppose a box contains two tiles, A and E. You plan to draw one tile out of the box.

   A. How many permutations are there of one draw from a box of two tiles? __________

   B. Change the **Number of draws from box** to 2. What are the different permutations when you draw 2 tiles out of 2? ____

   C. Look at the **TREE** tab. If you pick A first, what is the only possibility for the second draw? ______________________________

   D. If you pick E first, what is the only possibility for the second draw? _________

2. Change the **Number of tiles in box** to 3. The **Number of draws from box** should still be 2.

   A. How many choices are there for the first draw? ________

   B. Once you pick the first tile, how many choices are there for the second draw? ______

   C. How many permutations are there, total? __________

   D. How does the number of permutations relate to the number of choices for the first and second draws? ____

   ____

   E. Look at the **TREE** tab. How does the **TREE** tab represent the choices for the first and second tiles? ____

3. Check that there are three tiles in the box and the number of draws is two. Select the **NOTATION** tab. The number of permutations is represented by the symbolic notation $_{n}P_{r}$.

   A. In $_{n}P_{r}$, what does n stand for? ____

   B. In $_{n}P_{r}$, what does r stand for? ____

   C. What does $_{3}P_{2}$ equal? __________

(Activity A continued on next page)

Activity A (continued from previous page)

4. Change the **Number of tiles** to 4. Check that the **Number of draws** is still set to 2.

   A. How many choices are there for the first draw? __________

   B. How many choices are there for the second draw? __________

   C. What does ${}_4P_2$ equal? __________

   D. How do you calculate this result? ____

5. Suppose there were 4 tiles and 3 draws. (Do not change the Gizmo yet.)

   A. How many choices are there for the first tile? _________

   B. How many choices are there for the second tile? _________

   C. How many choices are there for the third tile? _________

   D. What does ${}_4P_3$ equal? __________ Confirm your answer with the Gizmo.

6. How many permutations are possible when drawing 3 tiles from a box of 5 tiles? Show your calculation. ${}_5P_3$ = _________________________ Check your answer using the Gizmo.

7. Suppose a box contains all 26 letters in the alphabet. How many permutations are there if you draw 4 letters? Show your calculation. ${}_{26}P_4$ = __________________________________

8. Challenge: Suppose you are drawing *r* tiles from a box of *n* letter tiles, and order matters.

   A. How many choices are there for the first tile? _________

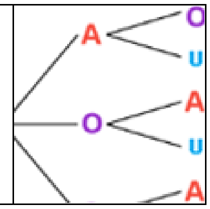
   B. How many choices are there for the second tile? _________

   C. How many choices are there for the third tile? _________

   D. How many choices are there for the r^{th} tile? (Careful!) __________

   E. Based on your answers to the questions above, write an expression for ${}_nP_r$.

   ${}_nP_r$ = _____________________________

Activity B: **Permutations and factorials**	<u>Get the Gizmo ready</u>: • Set the **Number of tiles in box** to 5. • Set the **Number of draws from box** to 5. • Check that **Yes** is chosen for **Is order important?** • Check that the **TREE** tab is selected.	

1. How many permutations are possible when drawing 5 tiles from a box of 5 tiles?

   Show your calculation: _______ • ______ • ______ • ______ • ______ = _________

   Check your answer by selecting the **NOTATION** tab.

2. The calculation you just completed can be rewritten in a shorter way using **factorials**. The factorial of an integer is the product of the integer and all the positive integers below it. For example, 5! = 5 • 4 • 3 • 2 • 1, or 120. (Note: By definition 0! = 1.)

   Using the factorial notation, how many permutations are possible when choosing *n* tiles from a box of *n* tiles? ${}_nP_n$ = ______________

3. In general, when drawing *r* tiles from a box of *n* tiles, if order matters, the number of possible permutations is:

$${}_nP_r = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-r+1)$$

   For example, the number of possible permutations when drawing 3 tiles from a box of 5 is ${}_5P_3 = 5 \cdot 4 \cdot 3 = 60$. This calculation can be rewritten as a ratio of factorials. (Notice that the factorial in the denominator cancels out the “tail” of the factorial in the numerator.)

$${}_5P_3 = 5 \cdot 4 \cdot 3 = \frac{5 \bullet 4 \bullet 3 \bullet \cancel{2 \bullet 1}}{\cancel{2 \bullet 1}} = \frac{5!}{2!}$$

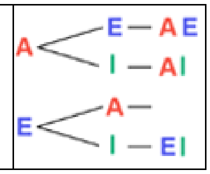
   A. Rewrite the following permutation calculations using factorial notation, as shown in the example above.

   ${}_6P_2$ = ${}_7P_3$ =

   B. In each fraction, how does the numerator relate to *n* and *r*? ____________________

   C. How does each denominator relate to *n* and *r*? ______________________________

   D. <u>Challenge</u>: Based on your answers to B and C, write a general formula for ${}_nP_r$ in factorial notation in the space to the right. ${}_nP_r$ =

Activity C: **Combinations**	Get the Gizmo ready: • Set the **Number of tiles in box** to 3. • Set the **Number of draws from box** to 2. • Next to **Is order important?** select **No**.	

Introduction: In some cases, the order of events or objects is important. For example, order matters when you are spelling words with letters. In other cases, order is not important. For example, a group of coins represents the same value no matter now they are ordered.

1. Select the **TREE** tab. Notice that some of the different permutations are grayed out. These are the permutations that duplicate other permutations. For example, AE and EA are two different permutations, but they are only counted as one combination.

    A. List the equivalent pairs of combinations: EA and ____ IA and_____ IE and ____

    B. How many combinations are there for 2 draws from a box of 3 tiles? ______________

    The notation for the number of possible combinations when you draw 2 tiles out of a set of 3 tiles is ${}_3C_2$. In general, the number of combinations resulting from choosing r objects from a set of n objects is given by the notation ${}_nC_r$.

2. Set the **Number of tiles in box** to 4 and the **Number of draws from box** to 1. Select the **LIST** tab. Use the **LIST** tab to find the number of possible permutations and combinations for each of the following situations, increasing the number of draws by 1 each time.

    ${}_4P_1$ = ________ ${}_4P_2$ = ________ ${}_4P_3$ = ________ ${}_4P_4$ = ________

    ${}_4C_1$ = ________ ${}_4C_2$ = ________ ${}_4C_3$ = ________ ${}_4C_4$ = ________

    A. In general, how does the number of combinations compare to the number of

    permutations? ____

    B. Notice that when r is equal to n, there is only one combination. Why is this true?

    ____

    ____

    C. Notice that, for each set of n and r values, ${}_nC_r$ is equal to ${}_nP_r$ divided by an integer. What are these integers?

    ${}_4C_1 = {}_4P_1 \div$ ______ ${}_4C_2 = {}_4P_2 \div$ ______ ${}_4C_3 = {}_4P_3 \div$ ______ ${}_4C_4 = {}_4P_4 \div$ ______

    D. How do these integers relate to the values of r? (Hint: Think about factorials.)

    ____

(Activity C continued on next page)

Activity C (continued from previous page)

3. If you are drawing 3 tiles from a box of 4 tiles, each combination of 3 tiles has 3! (6) permutations. For example, there are 6 permutations of A, E, and I: AEI, AIE, EAI, EIA, IAE, and IEA. If order does not matter, these 6 permutations count as just 1 combination. In other words, the number of combinations is equal to the number of permutations divided by 3!

   A. In general, how do you calculate the number of combinations, ${}_nC_r$, if you know the number of permutations, ${}_nP_r$? ____

   B. Select the **NOTATION** tab. What is the general formula for finding the number of combinations of *r* draws from a box of *n* tiles? ${}_nC_r$ = ________

4. In general, the number of permutations of *r* objects drawn from a set of *n* objects is given by:

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

   In the space to the right, derive a similar formula for the number of combinations from *n* tiles and *r* draws.

5. Use the formula to calculate the number of combinations for each of the following situations. Show your work. Use the Gizmo to check your answers.

   ${}_5C_2$ = _________ ${}_5C_4$ = _________

6. Amanda has 6 different coins (a penny, a nickel, a dime, a quarter, a half-dollar, and a silver dollar) in her pocket. She randomly picks 4 coins from her pocket.

   A. What are the values of *n* and *r* in this situation? *n* = _________ *r* = _________

   B. How many different permutations of four coins can she pick? _________

      Show your work to the right.

   C. How many different combinations of four coins can she pick? _________

      Show your work to the right.