Gizmos

Name: ______________________________ Date: ____________________

Student Exploration: Points in the Complex Plane

Vocabulary: additive inverse, complex conjugate, complex number, complex plane, imaginary unit, imaginary axis, imaginary number, quadratic formula, real axis, real number

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
The **quadratic formula**, shown to the right, is used to find the solutions of a quadratic equation in the form $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Use the quadratic formula to find the solutions of $x^2 - 3x + 2.5 = 0$. Show your work in the space to the right. Get as far as you can.

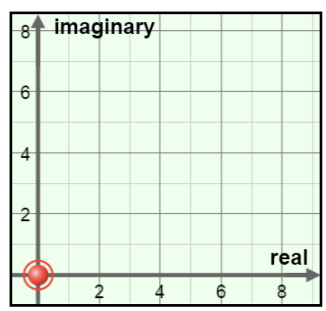
2. You should get $x = \dfrac{3 \pm \sqrt{-1}}{2}$ or $x = 1.5 \pm \dfrac{\sqrt{-1}}{2}$. What is the square root of –1? ____________

   Explain. ______________________________

3. Negative numbers do not have real square roots, but they do have roots that are **imaginary numbers**. The **imaginary unit** i is defined as the square root of –1: $i = \sqrt{-1}$ and $i^2 = -1$.

   Substituting i for $\sqrt{-1}$, what are the roots of $x^2 - 3x + 2.5$? ______________________________

Gizmo Warm-up
Although imaginary numbers may sound strange, they are useful in several branches of math and science. For example, the roots of some quadratics are **complex numbers** (z) that consist of a real part (a) and an imaginary part (bi): $z = a + bi$ (a and b are **real numbers**).



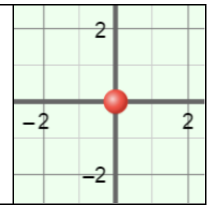
The *Points in the Complex Plane* Gizmo allows you to plot complex numbers on a grid called the **complex plane**. Notice that the complex plane has a horizontal **real axis** and a vertical **imaginary axis**.

1. Drag the red point into the origin at the center of the grid. Move the ***a*** slider back and forth.

   What happens to the red point? ______________________________

2. Return the point to the origin, and move the ***b*** slider back and forth. How does this affect the

   position of the red point? ______________________________

Activity A: The complex plane	Get the Gizmo ready: • With the red point on the graph, set ***a*** and ***b*** to 0. (To quickly set a slider to a value, type the value into the text box to the right of the slider, and hit **Enter**.)	

1. With the red point at the origin, move the ***a*** slider back and forth. Look on the chart below ***z = a + bi***. The value of the complex number ***z*** for each point is given on the chart.

   A. Set ***a*** to 1. What is the value of *z*? ____________

   B. Set ***a*** to –5. What is the value of *z* now? ____________

   When $b = 0$, the number is an ordinary real number and is plotted on the real axis.

2. Set ***a*** to 0 and ***b*** to 2.

   A. What is the value of *z*? ____________

   B. Set ***b*** to –8. What is the value of *z* now? ____________

   When $a = 0$ and $b \neq 0$, the number is imaginary and lies on the imaginary axis.

3. Set ***a*** to 4 and ***b*** to 7. What is the value of ***z***? ____________

   When $a \neq 0$ and $b \neq 0$, the number is a complex number of the form $z = a + bi$.

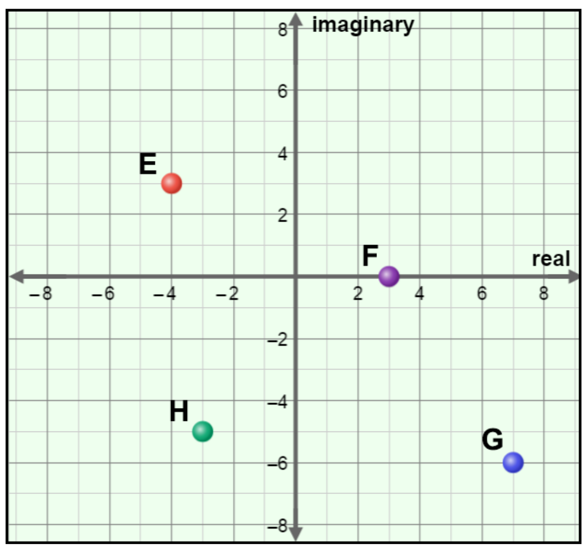
4. Plot the points A–D on the grid to the right. Then write the coordinates of points E–H below, using the *a* + *bi* notation. Use the Gizmo to check your answers.

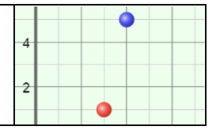
   A. $8 + 4i$ E. ____________

   B. $-5 - 2i$ F. ____________

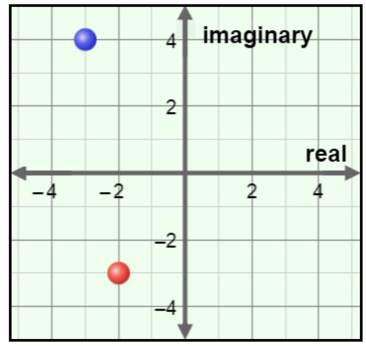
   C. $-2 + 7i$ G. ____________

   D. $-3i$ H. ____________



Activity B: **Complex arithmetic**	<u>Get the Gizmo ready</u>: • Remove all points from the grid. Drag the red point to 3 + *i* and the blue point to 4 + 5*i*.	

1. To add complex numbers, just add the real parts and then add the imaginary parts. For example, the sum of 2 + 3*i* and 4 + 5*i* is (2 + 4) + (3 + 5)*i*, or 6 + 8*i*.



   A. What is the sum of (3 + *i*) and (4 + 5*i*)? ____________

   B. Find the sum of –4 + 6*i* and 5 – 3*i*. ____________

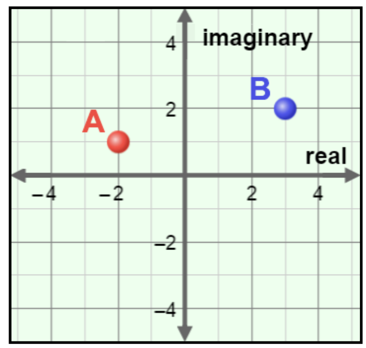
   C. What is the sum of the two complex numbers graphed to the right? Show your work below.

   ____________ + ____________ = ____________

2. The **additive inverse** of a complex number z_1 is a complex number z_2 where $z_1 + z_2 = 0$.

   A. In the Gizmo, drag the red point to 4 + 5*i* and a second point to its additive inverse.

   What is the additive inverse of 4 + 5*i*? ______________



   B. On the grid to the right, plot the additive inverses of points A and B. Label the additive inverses A’ and B’. How is the position of each additive inverse related to the position of the original point?

   ____

   ____

   C. In general, how do you find the additive inverse of a complex number? ____________

   ____

3. Remove all points but the red point, and place the red point at 3 + 5*i*. Turn on **Show complex conjugates**. The **complex conjugate** ($\bar{z}$) of a complex number has the same real part and the opposite imaginary part. For example, the conjugate of *a* + *bi* is *a* – *bi*.

   A. What is the complex conjugate of 3 + 5*i*? $\bar{z}$ = ___________

   B. Move the red point to –3 – 4*i*. What is the complex conjugate of –3 – 4*i*? ___________

   C. Drag the red point around. Where is the complex conjugate located, relative to the original point? ____

(Activity B continued on next page)

Activity B (continued from previous page)

4. Multiplying two complex numbers is similar to multiplying binomials. For example, the product of $(2 + 3i)$ and $(1 + 4i)$ is:

$$
\begin{aligned}
(2 + 3i)(1 + 4i) &= (2 \cdot 1) + (2 \cdot 4i) + (3i \cdot 1) + (3i \cdot 4i) \\
&= 2 + 8i + 3i + 12i^2 \\
&= 2 + 11i + 12(-1) \qquad \text{(Recall that } i^2 = -1\text{)} \\
&= 2 + 11i - 12 \\
&= -10 + 11i
\end{aligned}
$$

Find the following products. Show your work.

$(2 + 5i)(3 + i) =$ ____________ $(4 - i)(5 + 2i) =$ ____________

5. An interesting thing happens when you multiply a complex number by its conjugate. Try the following problems. Show your work.

$(4 + 2i)(4 - 2i) =$ ____________ $(-1 + 6i)(-1 - 6i) =$ ____________

What is true about the product of a complex number and its conjugate? ________________

____

6. Extension: Complex conjugates can help when dividing one complex number by another. For example, consider $(14 + 5i) \div (1 + 4i)$. First, write it as a fraction. Then, simplify by multiplying the numerator and the denominator by the conjugate of the denominator, $(1 - 4i)$.

Complete the calculation to find the quotient. See if you can simplify the answer to a single complex number.

$\dfrac{14 + 5i}{1 + 4i} \cdot \dfrac{1 - 4i}{1 - 4i} =$ ________________