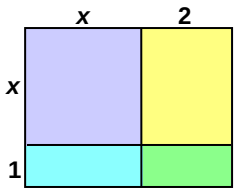
Name: _____ Date: _____

Student Exploration: Quadratics in Factored Form

Vocabulary: factored form of a quadratic function, linear factor, parabola, polynomial, quadratic function, root of an equation, vertex of a parabola, *x*-intercept

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. The sides of the large rectangle to the right measure ($x + 2$) and ($x + 1$).



   A. The rectangle has been divided into four regions. Label each region in the rectangle with its area.

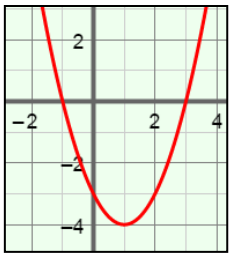
   B. What is the total area of the large rectangle? _____

   This **polynomial** is the product of the two **linear factors**, ($x + 2$) and ($x + 1$).

2. The area of another rectangle is $x^2 + 5x + 6$. If one side measures ($x + 2$), what is the measure of the other side? _____

Gizmo Warm-up

A function in which *y* depends on the square of *x* is a **quadratic function**. The graph of a quadratic function is a **parabola**, as shown to the right.



A quadratic function can be written in **factored form**: $y = a(x - r_1)(x - r_2)$. You will explore this type of quadratic function in the *Quadratics in Factored Form* Gizmo.

To begin, set ***a*** to 1. (Change the values of ***a***, r_1, or r_2 by dragging the sliders, or by clicking in the text field, typing in a value, and hitting **Enter**.)

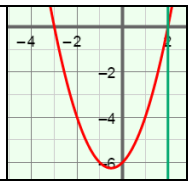
1. Turn on **Show *x*-intercepts**. Drag the r_1 and r_2 sliders to vary the values. Watch the values of the ***x*-intercepts** (the *x*-coordinates where the graph intersects the *x*-axis) as you do.

   How are r_1 and r_2 related to the *x*-intercepts? _____

2. Set ***a*** to 0, and then slowly drag the ***a*** slider to the right. What happens as *a* increases?

   _____

3. Set ***a*** to –1. What is true when *a* is less than zero? _____

Activity A: **The graph of** $y = a(x - r_1)(x - r_2)$	Get the Gizmo ready: • Turn off **Show *x*-intercepts**. • Turn on **Show probe**. • Set ***a*** to 1, r_1 to –3, and r_2 to 2.	

1. The function graphed in the Gizmo should be $y = (x + 3)(x - 2)$.

   A. What are the values of r_1 and r_2 for this equation? r_1 = _____ r_2 = _____

   B. Drag the probe to r_1 and then r_2. What is the y value at each of these points? _____

   C. Evaluate $y = (x + 3)(x - 2)$ at $x = r_1$ and then at $x = r_2$. Show your work below.

   D. Turn on **Show *x*-intercepts**. What happens when the function is evaluated at its x-intercepts? _____

   The x-intercepts are the **roots**, or solutions, of the related equation $(x + 3)(x - 2) = 0$.

   E. If the product of $(x - r_1)$ and $(x - r_2)$ is zero, what must be true about at least one of these factors? _____

   This is the *zero product property*.

2. With ***a*** set to 1, vary the values of r_1 and r_2 to graph different functions of the form $y = (x - r_1)(x - r_2)$. What is the value of $y = (x - r_1)(x - r_2)$ at r_1 and r_2? _____

3. Graph $y = 4(x - 1)(x + 5)$ in the Gizmo.

   A. What are the values of r_1 and r_2 for this function? r_1 = _____ r_2 = _____

   B. Why are r_1 and r_2 roots of the equation $4(x - 1)(x + 5) = 0$? _____

   _____

   C. Vary ***a*** to graph different functions of the form $y = a(x - 1)(x + 5)$. Does a have any effect on the roots? _____ Explain. _____

   _____

(Activity A continued on next page)

Activity A (continued from previous page)

4. Set ***a*** to 1. Vary the values of r_1 and r_2 to find several parabolas with only one *x*-intercept.

   A. What is the relationship between r_1 and r_2 when the graph has only one *x*-intercept?

   _____

   B. The **vertex of a parabola** is the maximum or minimum point of the parabola.

   When there is only one *x*-intercept, how are the vertex of a parabola and its *x*-intercept related? _____

   C. When $a = 1$, what is the factored form of a quadratic function with its vertex at the origin? _____ Check your answer in the Gizmo.

   D. While the vertex is on the *x*-axis, vary ***a***. What happens to the vertex and *x*-intercept?

   _____

   Experiment with a variety of functions to check that this is always true.

   E. Set ***a*** to 1. Vary the values of r_1 and r_2 to view a variety of parabolas with two *x*-intercepts. Where is the vertex located in relationship to the two *x*-intercepts?

   _____

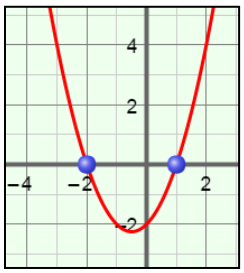
5. Find the quadratic function in factored form for each parabola described or shown below. Check your answers in the Gizmo by graphing your functions.

   A. *x*-intercepts –4 and 0, $a = 3$

   _____

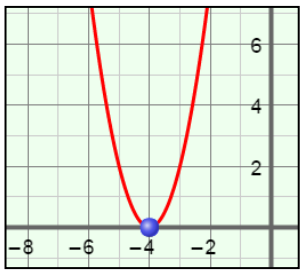
   C. *x*-intercepts –3 and 3, $a = –1$

   _____

   B. 

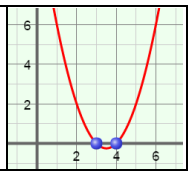
   $a = 1$

   _____

   D. 

   $a = 2$

   _____

Activity B: **Factored form and polynomial form**	Get the Gizmo ready: • Be sure **Show *x*-intercepts** and **Show probe** are turned off. • Set ***a*** to 1, $\boldsymbol{r_1}$ to 3, and $\boldsymbol{r_2}$ to 4.	

1. The function graphed in the Gizmo should be $y = (x - 3)(x - 4)$.

   A. You can multiply the right side of $y = a(x - r_1)(x - r_2)$ to write it in **polynomial form**, $y = ax^2 + bx + c$.

   Multiply $(x - 3)(x - 4)$ to write $y = (x - 3)(x - 4)$ in polynomial form. Show your work to the right. Then select **Show polynomial form** to check your answer.

   B. How can you combine r_1 and r_2 in the factored form to get b in the polynomial form?

   _____

   C. How can you combine r_1 and r_2 to get c? _____

   _____ Experiment with other functions to check that this is always true.

   D. Multiply $(x - r_1)(x - r_2)$ to write $y = (x - r_1)(x - r_2)$ in polynomial form. Show your work to the right.

   E. How does the "multiplied-out" version of $y = (x - r_1)(x - r_2)$ show how r_1 and r_2 can be used to find b and c in the polynomial form? _____

   _____

2. With ***a*** still set to 1, vary the values of $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ to find several parabolas with one x-intercept.

   A. How can you use the value of r_1 to get the value of c in the polynomial form?

   _____

   B. How can you use the value of r_1 to get the value of b in the polynomial form?

   _____

   C. If $a = 1$, how can you tell if a function written in polynomial form has exactly one x-intercept? _____

(Activity B continued on next page)

Activity B (continued from previous page)

3. Be sure **Show polynomial form** is still turned on.

   A. Use the Gizmo to help you fill in the table for each of the functions in the first column.

Factored form	Polynomial form when $a = 1$	Polynomial form when $a = 2$	Polynomial form when $a = -3$
$y = a(x - 2)(x - 4)$			
$y = a(x + 1)(x - 2)$			
$y = a(x - 5)(x + 2)$			

   B. How does *a* change the values of *b* and *c* in the polynomial form? _____

   _____

   C. Use r_1, r_2, and *a* in the blanks below to write equations that describe the relationships you discovered above.

   $b =$ _____ $c =$ _____

   D. Use the equations from above to fill the blanks below to write equations for the sum and product of the roots of a quadratic function.

   $r_1 + r_2 =$ _____ $r_1 r_2 =$ _____

4. One *x*-intercept of $y = x^2 - 6x + c$ is 3.

   A. How you can find the value of *c*? _____

   _____

   B. What is the value of *c*? _____

   C. What is true about the *x*-intercepts of this function? _____

5. One *x*-intercept of $y = x^2 + bx + 10$ is 5.

   A. How you can find the value of *b*? _____

   _____

   B. What is the value of *b*? _____

   C. What is the other *x*-intercept of this function? _____