Name: _____   Date: _____

Student Exploration: Roots of a Quadratic

Vocabulary: axis of symmetry, complex number, conjugates, discriminant, imaginary number, parabola, quadratic equation, quadratic formula, quadratic function, root of an equation

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)

1. Factor $x^2 + 3x + 2$ by filling in the blanks: $x^2 + 3x + 2 = (x +$ _____$)(x +$ _____$)$

    (Hint: The numbers in the blanks have a sum of 3 and a product of 2.)

2. What two values of x make the product above equal to zero? _____

3. Plug each of these values into $x^2 + 3x + 2$. What do you get? _____

Gizmo Warm-up
Quadratic functions are functions of the form $f(x) = ax^2 + bx + c$. The graph of a quadratic function is a **parabola**, as shown to the right. When working with quadratic functions, it is often useful to find the values of x that make $f(x)$ equal to zero.
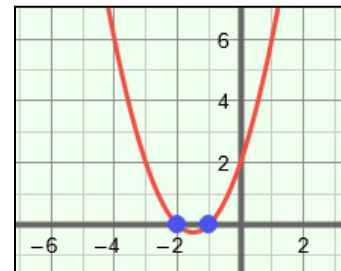
Factoring is one way to find these values, but factoring is not always easy. In the *Roots of a Quadratic* Gizmo, you will use algebraic and graphical methods to explore the values of x that make $f(x) = 0$.

To begin, graph $y = x^2 + 3x + 2$ by setting **a** to 1.0, **b** to 3.0, and **c** to 2.0. (Change the values by dragging the sliders, or by clicking in the text field, typing in a value, and hitting **Enter**.)

1. The blue points are *x*-intercepts of the parabola. They are the points where $y = 0$.

    Mouseover the blue points. What is the *x*-coordinate of each point? _____

2. The *x*-intercepts are solutions, or real **roots**, of the **quadratic equation** $x^2 + 3x + 2 = 0$.

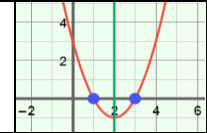   A. Plug each solution into $x^2 + 3x + 2$. What do you get? _____

    B. Recall that $x^2 + 3x + 2 = (x + 1)(x + 2)$. How do these factors relate to the *x*-intercepts

        of $y = x^2 + 3x + 2$? _____

Activity A: **Roots and the line of symmetry**	Get the Gizmo ready: • Set **a** to 1.0, **b** to –4.0, and **c** to 3.0.	

1. The roots of a quadratic equation are the values of x that make the related function zero. The real roots are also the *x*-intercepts of the parabola. Look at the graph of $y = x^2 - 4x + 3$.

    A. How many roots does $x^2 - 4x + 3 = 0$ have? _____ What are the roots? _____

    B. Change **c** to 4.0. How many roots does $x^2 - 4x + 4 = 0$ have? _____

2. Now graph $y = x^2 - 4x + 8$ in the Gizmo, and look at the resulting parabola. Do you think

    $x^2 - 4x + 8 = 0$ has any real roots? _____ Explain. _____

    _____

3. Vary the values of **a**, **b**, and **c**. In general, how many real roots are possible for a quadratic

    equation? _____

4. Graph $y = x^2 + 6x + 5$. Turn on **Show axis of symmetry x = –b/(2a)**. The **axis of symmetry** is a line that divides a parabola into two halves that are mirror images.

    A. How does the location of the axis of symmetry relate to the location of the two

    x-intercepts? _____

    B. Move the **a**, **b**, and **c** sliders. Which values affect the axis of symmetry? _____

    C. The equation of the axis of symmetry is $x = \dfrac{-b}{2a}$. How does this explain what you

    observed above? _____

5. Suppose you know the line of symmetry for a quadratic function.

    A. From just this information, can you find the *x*-intercepts? _____ Explain. _____

    _____

    B. Suppose the axis of symmetry of the graph of a quadratic function is at $x = 6$. If one

    root of the related quadratic equation is –1.5, what is the other root? _____

Activity B: The quadratic formula	Get the Gizmo ready: • Be sure the **CONTROLS** and **REAL PLANE** tabs are selected.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. Some quadratic equations are difficult to factor. In these cases, you can use the **quadratic formula**, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to find the roots of the quadratic equation $ax^2 + bx + c = 0$.

   A. Graph $y = 3x^2 - x - 4$. Select the **SOLUTION** tab to see how the quadratic formula is used to find the roots of $3x^2 - x - 4 = 0$. What are the roots? _____

   Click on the **CONTROLS** tab to check that these are the x-intercepts of the graph.

   B. Use the quadratic formula to find the roots of $2x^2 + x - 10 = 0$. Show your work in the space to the right. Then check your answer in the Gizmo.

2. The **discriminant** is the part of the quadratic formula that is under the radical, $b^2 - 4ac$. It provides useful information about the number of real roots of a quadratic equation. On the **CONTROLS** tab, turn on **Show discriminant computation**.

   A. Graph each quadratic function listed to the right in the Gizmo. Then state the number of real roots of the related equation $(ax^2 + bx + c = 0)$ and give the discriminant.

Function	Number of real roots	Discriminant
$y = x^2 + 6x + 9$		
$y = x^2 - 5x - 8$		
$y = x^2 - 4x + 6$		

   B. Vary **a**, **b**, and **c**, and watch how the number of real roots and the discriminant change. In general, how does the discriminant relate to the number of real roots?

   _____

   _____

   C. Why do you think the discriminant determines the number of real roots of a quadratic equation? _____

   _____

(Activity B continued on next page)

Activity B (continued from previous page)

3. A fraction with a sum in the numerator can be rewritten as the sum of two fractions with the same denominator. For example, $\dfrac{1+x}{2} = \dfrac{1}{2} + \dfrac{x}{2}$.

    A. In the space to the right, rewrite the quadratic formula as the sum of two fractions.

    B. What line does the first fraction describe? _____

    C. What is the second numerator the same as? _____

    D. Whenever a quadratic function has two x-intercepts, they are always equidistant from the axis of symmetry. Use your findings from above to explain why this is true.

_____

_____

4. Convert each of the following equations to the form $ax^2 + bx + c = 0$. Then find the discriminant of each equation, predict the number of real roots, and find the roots (if necessary, to the nearest hundredth). Use the Gizmo to check your work.

Equation	$x^2 - 4x = -4$	$3x^2 - 3 = -5x$	$7x^2 - 3x = 1$
$ax^2 + bx + c = 0$			
Discriminant			
Number of real roots			
Real roots			

5. A box has a length of x in., a width of $(10 - x)$ in., and a height of 8 in.

    8 in.
    x in.
    $(10 - x)$ in.
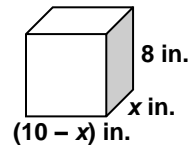
    A. What is the volume of the box? _____

    B. Karen wants to know if this box can have a volume of 192 in.^{3}. Write a quadratic equation that describes this situation. _____

    C. Use the discriminant to determine if the box can have a volume of 192 in.^{3}. Then, if possible, use the quadratic formula find the length and width of the box. Show your work in the space to the right.

	Get the Gizmo ready:
Activity C: **Complex roots**	• Be sure the **REAL PLANE** tab is selected. • Select the **CONTROLS** tab.

In some cases, the discriminant is negative. The roots of a quadratic equation with a negative discriminant are **complex numbers** of the form $a + bi$, where a and b are real numbers, and bi is an **imaginary number**. The i in the imaginary number is equal to $\sqrt{-1}$, so $i^2 = -1$.

1. Graph $y = 3x^2 + 4x - 6$ in the Gizmo. Slowly drag the **c** slider to the right. Watch the solutions below the sliders.

    A. What is true about the parabola when there are no real roots? _____

    _____

    B. Be sure the quadratic equation shown has two imaginary roots. Click on the **SOLUTION** tab to see how the solution is found using the quadratic formula.

    Why are the solutions complex numbers? _____

    _____

2. The square root of a negative number, such as –36, can be rewritten as the product of the square root of a positive number multiplied by $\sqrt{-1}$, or i. So, $\sqrt{-36} = \sqrt{36} \cdot \sqrt{-1}$, or $6i$.

    A. On the **CONTROLS** tab, graph $y = x^2 + 2x + 5$. What is the discriminant ($b^2 - 4ac$) of

    the quadratic equation $x^2 + 2x + 5 = 0$? _____

    B. What is the square root of this discriminant? _____

    C. Use the quadratic formula to find the two complex solutions. Show your work to the right. When you are done, check your solutions with the Gizmo.

3. Two complex numbers with the same real parts and opposite imaginary parts are called **conjugates**. Experiment with different quadratic equations with complex roots.

    A. Do complex roots always seem to occur in conjugate pairs? _____

    B. What parts of the quadratic formula result in the real and imaginary part of the roots?

    Real: _____   Imaginary: _____

(Activity C continued on next page)

Activity C (continued from previous page)

4. Select the **COMPLEX PLANE** tab above the graph to view the plane that complex numbers can be graphed on. To graph a complex number on the complex plane, graph the real part on the horizontal axis and graph the imaginary part on the vertical axis. So, to graph $a + bi$ on the complex plane, plot the point (a, bi).

   A. Graph the function $y = x^2 - 4x + 7$ in the Gizmo. What are the solutions of the related

   quadratic equation $x^2 - 4x + 7 = 0$? _____

   B. Look at the solutions plotted on the complex plane. Mouseover the blue points. What

   are the coordinates of the points?   (_____, _____) and (_____, _____)

   C. How do the coordinates correspond to the parts of the complex solutions? _____

   _____

   D. Switch back and forth between the **REAL PLANE** and **COMPLEX PLANE** tabs. How does the real part of both complex solutions relate to the parabola on the real plane?

   _____

   E. With the **COMPLEX PLANE** tab selected, vary the sliders until the solutions lie on the real axis of the complex plane. Select the **REAL PLANE** tab.

   What do you notice about the solutions in this case? _____

   _____

5. Find the complex roots of each of the following quadratic equations, to the nearest hundredth if necessary. Show your work. Then check your answers in the Gizmo.

   A. $x^2 + 36 = 0$ C. $2x^2 - 5x + 7 = 0$

   B. $x^2 + 6x + 12 = 0$ D. $4x^2 + 3x + 8 = 0$