Name: Date:

**Student Exploration: Rotations, Reflections, and Translations**

**Vocabulary:** image, preimage, reflection, rotation, transformation, translation

**b**

**Prior Knowledge Questions** (Do these BEFORE using the Gizmo.)

Little Johnnie is playing with a puzzle piece shaped like the letter “b.” He is surprised to discover he can make other letters with the same piece.

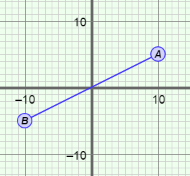
1. If he flips the “b” over to the left, what new letter is formed?

Draw a picture to the right.

1. If he flips the letter you drew above down, what new letter is formed?

Draw a picture to the right.

1. Little Johnnie puts the “b” back in its original position. Then he rotates it 90° clockwise. Draw the result to the right. Is this a letter?

**Gizmo Warm-up**

The manipulations of the letter “b” are examples of different **transformations** – **rotations** (turns), **reflections** (flips), and **translations** (slides). In the *Rotations, Reflections, and Translations* Gizmo™, you will rotate, reflect, and translate various figures on a coordinate plane. To begin, select **Segment** from the **Figure type** menu and **Rotate around Origin** from the **Operation** menu.

1. Drag the **Rotation (in degrees)** slider. What happens to ?

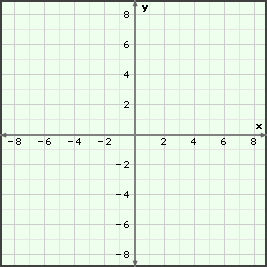
In all of the transformations in this Gizmo,  is the **image** and  is the **preimage**.

1. Select **Reflect over *x*-axis**. Drag points *A* and *B* so they are both above the *x*-axis. What do you notice about ?
2. Select **Translate**. Drag the ***x* translation** and ***y* translation** sliders. What happens to ?

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| **Activity A:**  **Translations** | Get the Gizmo ready:   * Under **Figure type**, select **Point**. * Under **Operation**, select **Translate**. |  |

1. Recall that point *A* is the preimage, and point *E* is the translated image of point *A*.
2. With the ***y* translation** slider set to 0, drag the ***x* translation** slider. How does this affect point *E*?
3. Now set the ***x* translation** slider to 0 and drag the ***y* translation** slider. How does this affect point *E*?
4. Set both translation sliders to a positive value. Drag point *A* around. How does this affect point *E*?
5. Turn on **Show table**. Set ***x* translation** to –5 and ***y* translation** to 6. Drag point *A* to (–2, 3).
6. What are the coordinates of point *E*? ( , )
7. How can you calculate the coordinates of point *E*?

1. Suppose a point has coordinates (*x*, *y*). What are the coordinates of the image if the *x* translation is *a* and the *y* translation is *b*? ( , )
2. The endpoints of  are at *A*(–5, 6) and *B*(4, 0). Predict the endpoints of the image  for the translations listed in the table below. Then sketch  and each image on the grid. Click on **Show table** to check your answers.



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| ***x* and *y***  **translation** | **Point *E***  **Image of *A*(–5, 6)** | **Point *F***  **Image of *B*(4, 0)** |
| *x* translation: 3  *y* translation: 0 |  |  |
| *x* translation: –1  *y* translation: –5 |  |  |
| *x* translation: 1  *y* translation: –6 |  |  |

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| **Activity B:**  **Reflections** | Get the Gizmo ready:   * Under **Figure type**, select **Point**. * Under **Operation**, select **Reflect over *x*-axis**. * Turn off **Show table**. |  |

1. Recall that point *E* (the image) is the reflection of point *A* (the preimage).
2. Drag point *A* up, down, left, and right. Fill in the table to describe how point *E* moves when you do this.

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| **Point *A*** | Up | Down | Left | Right |
| **Point *E*** |  |  |  |  |

1. Turn on **Show table**. Watch the coordinates in the table as you drag point *A* around. How do the coordinates of point *E* compare to the coordinates of point *A*?

1. A point has coordinates (*x*, *y*). What are the coordinates of the image if (*x*, *y*) is reflected over the *x*-axis? ( , )
2. Turn off **Show table**. Select **Reflect over *y*-axis**.
3. What do you think will happen to point *E* when point *A* is moved to the right?

1. Turn on **Show table**. How do the coordinates of point *E* compare to those of point *A*?

1. A point has coordinates (*x*, *y*). What are the coordinates of the image if (*x*, *y*) is reflected over the *y*-axis? ( , )
2. Under **Operation**, select **None**. Under **Figure type**, select **Triangle**. Drag the vertices of Δ*ABC* to *A*(7, 5), *B*(–10, 2), and *C*(2, –8).Predict the coordinates of the vertices of the image Δ*EFG* for the reflections listed below. Then check your answers in the Gizmo.

Over the *x*-axis: *E*( , ) *F*( , ) *G*( , )

Over the *y*-axis: *E*( , ) *F*( , ) *G*( , )

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| **Activity C:**  **Rotations** | Get the Gizmo ready:   * Under **Figure** type, select **Point**. * Under **Operation**, select **Rotate around Origin**. * Turn off **Show table**. |  |

1. Drag point *A*, the preimage, to (10, 5).
2. Drag the **Rotation (in degrees)** slider. What shape does point *E*, the image of *A*, trace as you drag the slider to the right?
3. Does point *E* move in a clockwise or counterclockwise direction when you drag the slider to the right?

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| **Point *E***  **Image of *A*(10, 5)** | **Angle of rotation** |
| (–5, 10) |  |
| (–10, –5) |  |
| (5, –10) |  |

1. Give the angle of rotation that places point *E* at the coordinates listed in the table to the right. Click on **Show table** to check your answers.
2. What angle of rotation brings point *E* back to point *A*?
3. Turn off **Show table**. Select **Segment** under **Figure type**. Set **Rotation (in degrees)** to 0°. Place the endpoints of  at *A*(–1, 10) and *B*(8, –3).
4. Predict the endpoints of the image  for each angle of rotation listed in the table below. Click on **Show table** to check your answers.

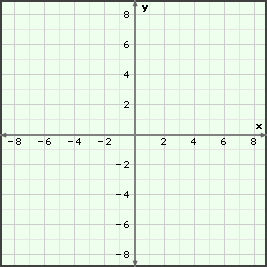
|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Angle of rotation** | **0°** | **90°** | **180°** | **270°** | **360°** |
| **Point *E***  **Image of *A*(–1, 10)** |  |  |  |  |  |
| **Point *F***  **Image of *B*(8, –3)** |  |  |  |  |  |

1. Based on the patterns you have observed, write the general coordinates of the image of a point with coordinates (*x*, *y*) in the table below.

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| **Angle of rotation** | **0°** | **90°** | **180°** | **270°** | **360°** |
| **Coordinates of image of (*x*, *y*)** |  |  |  |  |  |

**(Activity C continued on next page)**

**Activity C (continued from previous page)**

1. Suppose Δ*ABC* has the vertices listed below. Write the coordinates of the vertices of image Δ*EFG* for a 270° rotation in the table. Sketch both triangles on the grid to the right. Then check your answers in the Gizmo.

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| **Vertices of Δ*ABC*** | **Vertices of Δ*EFG*** |
| *A*(2, 3) |  |
| *B*(–5, 1) |  |
| *C*(4, –6) |  |

1. Extension: Select **Point** under **Figure type** and **Rotate around Origin** under **Operation**. Be sure **Show table** is turned on.
2. You can use the cosine (cos) and sine (sin) of the angle of rotation to find the image of any point on the positive *x*-axis. To see how this works, first use a calculator to find the cosine and sine (to the nearest hundredth) of the angles in the table below.

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| **Angle of rotation (*θ*)** | **cos *θ*** | **sin *θ*** | **Point *E***  **Image of *A*(10, 0)** |
| 30° |  |  |  |
| 45° |  |  |  |
| 60° |  |  |  |
| 90° |  |  |  |

1. Drag point *A* to (10, 0). Find the coordinates of the image point *E* for each angle of rotation listed above. Write the coordinates of point *E* in the last column of the table.
2. Compare the cosine and sine for each angle to the coordinates of point *E*. What do you notice?

Experiment with other points on the positive *x*-axis and other angles to verify.

1. The point (*x*, 0) is rotated through an angle of *θ* when *x* > 0. Based on the patterns you have observed, use cosine (cos) and sine (sin) to write a formula for the coordinates of the image.

( , )