Name: ______________________________________ Date: ________________________

Guided Learning: Reporting Measurements

Learning goals
After completing this activity, you will be able to …
- Explain the differences between accuracy, precision, and resolution.
- Identify and count the significant digits in a number.
- Record data with the correct number of significant digits.
- Describe possible causes of error in an experiment.
- Calculate absolute error and percent error.

Vocabulary: absolute error, accuracy, percent error, precision, resolution, significant digits

Warm-up questions:
Arturo's father, Danilo, runs an Italian restaurant famous for its hearty minestrone soup. One day, Danilo is sick and asks Arturo to make the soup based on an old family recipe, shown at right. When he starts to make the soup, however, Arturo has trouble figuring out exact amounts.

Danilo's Famous Minestrone

1. **Fill pot with vegetable stock**
2. **Add 5 cans of diced tomatoes**
3. **Add 3 cans of navy beans**
4. **Add 10 carrots, chopped**
5. **Add 6 onions, chopped**
6. **Add ¼ cup spices (sage, thyme, basil, oregano)**
7. **Cook for 4-6 hours**
8. **Add 5 cups macaroni**
9. **Add 2 zucchinis, chopped**
10. **Add 1 cup grated parmesan**
11. **Add salt and pepper to taste**

1. The recipe says to fill a pot with vegetable stock. Why might this be a problem for Arturo?

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2. The recipe asks for 5 cans of tomatoes. Arturo finds 8-ounce cans and 16-ounce cans. What should he do?

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3. What other parts of the recipe might be difficult for Arturo to follow? Explain your thinking.

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Errors in data

If Arturo wanted to make a minestrone soup exactly like his father's, he might have some problems because the directions were not very detailed. How much vegetable broth should Arturo use? What size of tomato cans did the recipe call for? How much salt and pepper to add? Should the soup be cooked over high or low heat?

Just as changing the amount of an ingredient like salt could ruin the flavor of soup, small measurement errors can have major consequences in science. For example, a small velocity change may spell the difference between a satellite finding a stable orbit or crashing back to Earth. Other times, small errors may multiply. Measuring the angle to a target one degree wrong may not seem like much, but it could result in missing the target by a great distance if the target is many kilometers away.

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Martha and Max are each measuring the density of a small pebble. (Density is equal to mass divided by volume.) Martha measures the mass of the pebble as 19 grams and gets a volume measurement of 1 cm^3. Max measures the mass as 20 grams and the volume as 2 cm^3.

1. What value will each person calculate for the density of the pebble?

   Martha: _______ g/cm^3 Max: _______ g/cm^3

2. Each of Martha and Max's measurements were different by one unit. Why do you think the calculated density values were so different?

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3. Did the measurement error for mass or volume have a greater effect on the density calculation? Explain.

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4. Martha claims that the pebble likely contains solid gold because it is so dense. (Gold has a density of 19.6 g/cm^3.) Would you believe her? Explain why or why not.

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Accuracy, precision, and resolution

When scientists make measurements, record data, or give instructions, they are concerned with three characteristics: **accuracy**, **precision**, and **resolution**. Although these words are often confused for one another, they have distinct definitions:

- Accuracy refers to how close a measurement is to the actual value.
- Precision refers to the consistency of the measurement.
- Resolution refers to the fineness of the measuring device.

In other words, a measurement is accurate if it is close to the actual value. A set of measurements is precise if all of the measurements are similar. The resolution of a measuring device is high if it can detect very small changes in value.

The difference between accuracy and precision can be illustrated by a target. In this analogy, the center of the target represents the actual value. As you can see, it is possible for measurements to be accurate without precision or precise without accuracy.

Accurate but not precise

Precise but not accurate

Accurate and precise

Resolution is a quality of the measuring device. For example, a ruler that is marked in millimeters has a greater resolution than a ruler marked only in centimeters. Graduated cylinders are often marked in milliliters while flasks and beakers are often marked in 50-mL intervals. Because graduated cylinders have finer markings, they have greater resolution and can be used to obtain a measurement that is both more accurate and more precise than the measurement made using a beaker.

1. Two students are measuring the volume of a glass of water. The actual volume of the water is 114.781 mL. Each student measures the water three times:

   Alan: 116.8 mL, 116.8 mL, 116.7 mL   Becky: 115.4 mL, 114.1 mL, 114.9 mL

   A. Who measured the water with the greatest precision? _______________

      Explain: ____

   B. Who measured the water with the greatest accuracy? _______________

      Explain: ____

2. Margot wants to measure exactly 5 grams of a reagent. Which device is most likely to give the most accurate measurement? Circle the answer and explain your choice.

   A. Double pan balance with 5-gram masses.

   B. Double beam balance with a 100-g rider and a 10-g rider.

   C. Triple beam balance with a 100-g rider, a 10-g rider, and a 1-g rider.

   D. Digital scale that reports mass to the nearest 0.001 g.

   Explanation: ____

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Significant digits

Mark measures the volume of water in a beaker marked in 50-mL intervals and reports the volume as 43.92 mL. Sherri measures a line with a ruler that has millimeters marked and reports the length as 5 centimeters. Although both of these measurements may be relatively accurate, they are also potentially misleading. Mark makes it sound as though he can detect volume differences of 0.01 mL. In contrast, a person reading Sherri's lab report would have no idea that she used a ruler capable of measuring millimeters.

Ideally, reported measurements should reflect the resolution of the instrument used to obtain the data. This is done by specifying the number of **significant digits** in the data. A significant digit is any value that has been directly measured. Other digits are not significant. Here are some rules for determining if a digit is significant or not:

- Any non-zero digit is significant: *227.4 has four significant digits.*
- Any digit that is between other significant digits is significant: *200.08 has five significant digits.*
- Zeros to the right of a significant digit *and* to the right of a decimal point are always significant: *6.00 has three significant digits.*
- Zeros used to space a number to the right of a decimal point are *not* significant: *0.000147 has only three significant digits.*
- Zeros to the right of a significant digit but to the left of a decimal point may or may not be significant: *875,000 has at least three significant digits, but may have as many as six.*
- When using scientific notation, all digits are significant: *8.75×10^5 has 3 significant digits.*

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1. Why might it be a problem if too many significant digits are reported for a measured value?

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2. Underline the significant digits of each of the numbers below. Circle digits that may or may not be significant. Write the number of definite significant digits next to each value. The first is done for you as an example.

0.000517 ___3__	1.000517 ______	0.004700 ______
83,000 ______	83,000.0 ______	1,000,000.00008 ______

Using significant digits

When you make a measurement, the data you record reflects the resolution of your instrument. A good rule of thumb is that you are allowed a maximum of one significant digit beyond the finest resolution of the instrument. For example, if you are using a graduated cylinder that is marked in milliliters, you are allowed to estimate the volume to the nearest 0.1 mL if you wish, but not to the nearest 0.01 mL. Do not record any significant digits beyond the instrument's finest resolution if you cannot make a good estimate. For example, when measuring a line with a ruler marked in millimeters, it is usually adequate to report the length to the nearest millimeter.

If a calculation involves measured values, use the following rules:

- When multiplying or dividing two measured values, the answer should have the same number of significant digits as the measurement with the fewest significant digits.

  Example: A rock has a mass of 210.6 g (four significant digits) and a volume of 80.3 cm^3 (three significant digits). Its density is 2.62 g/cm^3 (three significant digits).

- When multiplying a measured value by a whole number, the answer should have the same number of significant digits as the measured value. (You can assume the whole number has infinite significant digits.)

  Example: A marble has a mass of 65.4 g (three significant digits.) The mass of six marbles is 392 g (three significant digits).

- When adding two measured values, the answer should have the same number of *decimal places* (not significant figures) as the value with the fewest decimal places.

  Example: A rock has a volume of 21.0 cm^3 (one decimal place). A second rock has a volume of 4.52 cm^3 (two decimal places). The total volume of the two rocks is 25.5 cm^3 (one decimal place).

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1. Besides the value itself, what additional information can you get from a measurement?

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2. Calculate each of the following results, and report the answer with the correct number of significant digits. Use the equations below to help.

average speed = distance ÷ time   density = mass ÷ volume

volume of a box = length × width × height   pressure = force ÷ area

A. A horse travels 54 meters in 5.6 seconds. What is its average speed? ________

B. What is the volume of a box if *l* = 4.53 cm, *w* = 6.71 cm, and *h* = 3.15 cm? ________

C. What is the density of a brick with *m* = 1,005 g and *V* = 478.6 cm^3? ________

D. How much pressure does a 40,172-N circus elephant exert on a platform that has an area of 9,255 cm^2? ________

Uncertainty and error

Errors can affect the accuracy and precision of any experiment. Some problems are caused by human error, some from faulty equipment, and others from subjects that are variable or difficult to measure.

When the actual or theoretical value is known, it can be compared directly to the measured value. The **absolute error** of a measured value is equal to the difference between the actual value and the measured value:

absolute error = measured value – actual value

(Note: In this equation, you may substitute "theoretical value" for actual value if only the theoretical value is known.)

Mark measures the mass of a rock as 686 g, while the actual mass is 683 g. Maya measures the mass of a pebble as 6 g when its actual mass is 3 g.

1. What is the absolute error of each measurement? Mark: __________ Maya: __________

2. In your opinion, were both measurements equally accurate? Explain. __________________

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Percent error

In many cases, the absolute error is not very useful for comparing the accuracy of two calculations. For example, suppose an astronomer measures the Earth-Moon distance and gets a value of 384,400 km, which is 1 km different from the actual average distance of 384,399 km. Meanwhile, a student measures the height of a tree and gets a value of 14 meters for a tree that is actually 11 meters high. While the student's error of 3 meters was far smaller than the astronomer's error of 1,000 meters, most people would not agree that the astronomer's measurement was less accurate than the student's measurement. In these situations, it is useful to calculate the **percent error** (% error) or the percentage difference between the measured value and the actual value:

$$\%\, error = \left(\frac{measured - actual}{actual} \right) \times 100$$

The percent error of the astronomer's measurement of the Moon's distance is 0.00026%, while the percent error of the student's measurement of the tree is 27%. Thus, based on percent error, the astronomer's measurement was much more accurate than the student's measurement.

1. At the doctor's office, the nurse measures your height as 156.0 cm. If your actual height is 153.1 cm, what is the percent error of this measurement? _________

2. Based on the equation for percent error, what does it mean if the percent error is negative?

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3. **Critical thinking:** Why is percent error usually a more useful value than absolute error?

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