Name: ______________________________________ Date: ________________________

Guided Learning: Mass-Energy Equivalence

Learning goals

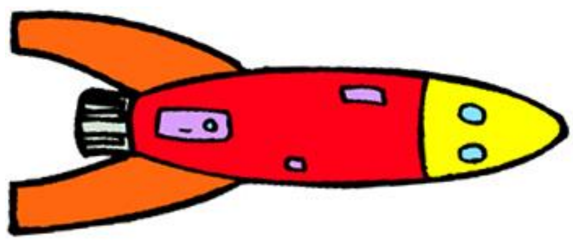
After completing this activity, you will be able to …

- Calculate time dilation, length contraction, and relativistic mass for a moving object.
- Explain mass-energy equivalence.
- Describe why mass-energy equivalence causes mass defect.
- Define nuclear fusion and fission.
- Give examples of applications of nuclear fusion and fission.

Vocabulary: binding energy, fission, fusion, length contraction, mass defect, mass-energy equivalence, nuclear reaction, nucleon, relativistic mass, rest mass, special relativity, time dilation

Warm-up questions

The theory of **special relativity** states that no object can exceed the speed of light. Suppose you are on a superfast rocket ship that is traveling close to the speed of light relative to an observer on Earth. The rocket engine generates a constant force *F* that causes the rocket ship to accelerate forward.



1. As the rocket ship approaches the speed of light (which it cannot exceed), what must happen to the acceleration of the rocket ship? Explain your answer.

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2. Newton's second law states that force is equal to mass times acceleration: $F = ma$.
If the force is constant and the observed acceleration of the rocket decreases, what must happen to the observed mass?

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As you have probably inferred, the measured mass of the rocket ship actually increases as its speed relative to the observer increases. This mass is known as **relativistic mass** (m') which can be distinguished from the original mass, or **rest mass** (m) of the body.

Taking relativistic mass into account, Newton's second law can be modified to say $F = m'a$.

Activity A: Calculating time, length, and mass

Special relativity states that three things happen when an object's speed approaches the speed of light relative to a stationary observer: clocks on the object appear to run more slowly to the observer (**time dilation**), the measured length of the object decreases (**length contraction**), and the relativistic mass of the object increases. The equations for these effects are similar:

Time dilation: $$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

t' is the time on the stationary frame
t is the time on the moving object
c is the speed of light

Length contraction: $$L' = L\sqrt{1-\frac{v^2}{c^2}}$$

L' is the measured length of the moving object
L is the original length (rest length) of the moving object

Relativistic mass: $$m' = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$$

m' is the measured mass (relativistic mass) or the moving object
m is the original mass (rest mass)

Notice that all of these equations feature the term $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, or γ (gamma).

Thus the three equations can be written more simply as $t' = \gamma t$, $L' = L / \gamma$, and $m' = \gamma m$. (Note: Because γ contains a speed squared divided by a speed squared, γ itself has no units.)

Calculator recommended

A rocket ship with a rest length of 20 m and a rest mass of 16,000 kg passes Earth moving at a speed of 0.85*c*. The rocket ship has a large clock on its side. An observer on Earth films the rocket ship as it passes.

1. Given the speed of the rocket ship, what is the value of γ? _______________

   (Hint: Use 0.85 for *v* and 1.00 for *c*.)

2. The observer measures the rocket ship's clock tick 1 second. How many seconds passed for the observer in this time interval? _______________

3. How long does the rocket ship appear to be to the observer? _______________

4. From the perspective of the observer, what is the mass of the rocket ship? _______________

   (Note: The observer could determine the rocket ship's relativistic mass if she knows the force its engine exerts and measures its acceleration.)

Activity B: Mass and energy

If an object increases its mass and velocity as it is pushed by a force, its kinetic energy (*KE*) increases as well. The formula for kinetic energy is $KE = \frac{1}{2} mv^2$.

Given the equation for the relativistic mass of a moving object, Einstein found that the change in relativistic mass ($\Delta m'$) is equal to the kinetic energy of the object divided by the square of the speed of light:

$$\Delta m' = \frac{1}{2} m'v^2/c^2$$

$$\Delta m' = KE/c^2$$

Einstein then realized that this relationship held true for *any* type of energy, not just kinetic energy. By rearranging the terms of the equation above, replacing kinetic energy (*KE*) by energy (*E*) and replacing change of mass ($\Delta m'$) by mass (m'), Einstein created the famous **mass-energy equivalence** equation:

$$E = m'c^2$$

Note that in applications where the object is not moving, the relativistic mass (m') is equal to the rest mass (*m*) and this equation is reduced to its most familiar form, $E = mc^2$.

So what does this equation mean? At a fundamental level, it means that mass and energy are two ways of describing the same property. Increasing an object's energy is equivalent to an increase in its mass. Doing work to an object by moving it or heating it up will increase both the object's energy and its mass. For example, a heated bar of gold has a tiny bit more mass than a cold bar of gold.

Because of the c^2 term in the equation (recall the *c* is the speed of light) a small amount of mass is equivalent to an enormous amount of energy. For example, a kilogram of mass is equivalent to 8.988×10^{16} J or 89,880,000,000,000,000 joules of energy. This much energy could provide electricity to a small country for a year!

In most applications, particles are not destroyed to produce pure energy, and outside of the quantum realm energy is not crystallized to form new particles. Instead, the mass of a particle or group of particles may change by altering their speed or positions, releasing or absorbing energy in the process. These processes are most important in **nuclear reactions**, or changes to the nuclei of atoms.

Enormous energy is released in the detonation of a hydrogen bomb.

1. In the equation $E = mc^2$, what does the *E*, *m*, and *c* stand for?

E _______________ *m* _______________ *c* _______________

2. What is the significance of $E = mc^2$? ____

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Activity C: Mass defect and binding energy

Mass-energy equivalence implies that as the energy of an object is increased, its mass increases as well. One way to increase the energy of an object is to increase its potential energy by doing work on the object against an attractive force. For example, if you lift a book to a higher shelf, you increase its gravitational potential energy. Because the potential energy of the book increases, its mass also increases by a tiny amount equal to the change in potential energy divided by the speed of light squared: $\Delta m = \Delta GPE \div c^2$. In everyday life, this increase in mass is so tiny it can be ignored. For example, the change in mass associated with lifting a 1-kg object 10 meters is about 1.09×10^{-12} grams, or 0.00000000000109 g.

Protons and neutrons in the nucleus of atoms are bound by the strong force. Energy must be supplied to overcome the strong force and pull protons and neutrons apart. The process of separating protons and neutrons increases their potential energy and thus increases their mass. Therefore, individual unbound protons and neutrons have a greater mass than protons and neutrons that are held tightly by the strong force in nuclei. The difference between the mass of a nucleus and the mass of an equivalent number of unbound **nucleons** (protons and neutrons) is the **mass defect** of the nucleus. By multiplying the mass defect by the speed of light squared, the mass defect can be converted to the **binding energy** of the nucleus, or the energy required to separate the nucleus into individual particles.

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1. What effect does increasing an object's potential energy have on its mass? Explain.

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2. How is the mass defect related to the strong nuclear force? ___________________________

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3. The mass of an unbound proton is 1.007276 universal mass units (u). The mass of an unbound neutron is 1.008665 u.

   For each of the atomic nuclei described below, calculate the mass of an equivalent number of unbound particles. Then subtract the actual atomic mass from the equivalent mass to find the mass defect. Leave the last column blank for now.

Isotope	Description	Atomic mass (u)	Equivalent mass (u)	Mass defect (u)	Defect per nucleon
Helium-4	2 p, 2 n	4.0026 u			
Carbon-12	6 p, 6 n	12.000 u			
Silicon-28	14 p, 14 n	27.977 u			
Iron-56	26 p, 30 n	55.935 u			
Silver-107	47 p, 60 n	106.91 u			
Gold-197	79p, 118 n	196.97 u			
Uranium-238	92 p, 146 n	238.05 u			

4. What trend do you see in the mass defect as the number of nucleons (protons and neutrons) increases? ____

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5. It is useful to consider the mass defect per nucleon. To do this, divide each mass defect by the number of nucleons (protons plus neutrons). Fill in the last column of the table.

   A. What pattern do you notice in the mass defect per nucleon from helium to iron?

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   B. What pattern do you notice in the mass defect per nucleon from iron to uranium?

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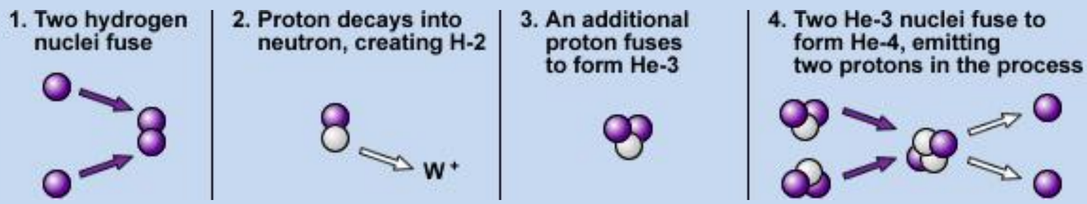
Activity D: Fission and fusion

As you have seen, the mass defect per nucleon increases at first, but then decreases as elements get heavier. Iron-56 has the greatest mass defect per nucleon of any element. That means it requires more energy to break apart an iron-56 nucleus than it requires for any other element. (Recall that the mass defect per nucleon is proportional to the average energy required to pry a nucleon from the nucleus.) If a nucleus with a small mass defect per nucleon (small binding energy) is changed to a nucleus with a large defect per nucleon (large binding energy), the difference in mass will be released as energy.

Nuclear reactions can either increase the mass of the nucleus or decrease the mass of the nucleus. In a **fusion** reaction, two nuclei smash together with such force that they merge together to form a heavier nucleus. In a **fission** reaction, a single unstable nucleus is split apart into two lighter nuclei.

Nuclear fusion

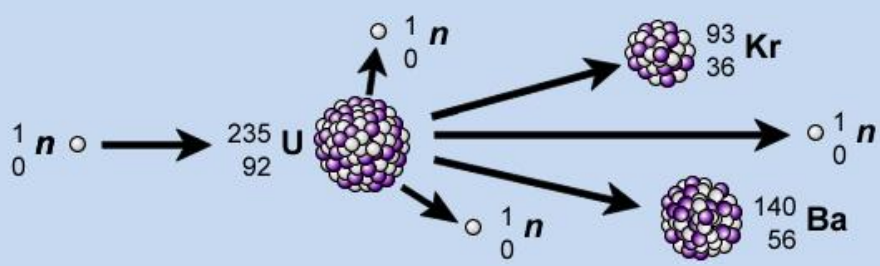
The fusion of hydrogen into stable helium-4 is a complicated process that takes several steps. First, two hydrogen nuclei must fuse together to form an unstable He-2 nucleus. Next, a proton decays into a neutron, emitting a W^+ boson. An additional proton fuses to the nucleus, forming unstable He-3. Finally, two He-3 nuclei fuse to form a stable He-4 nucleus. The two extra protons are emitted.



Nuclear fusion takes place in stars and in hydrogen bombs. For many years scientists and engineers have sought ways to harness the enormous power of fusion reactions to for electrical generation, but the enormous temperatures required for fusion to take place have so far defeated their efforts.

Nuclear fission

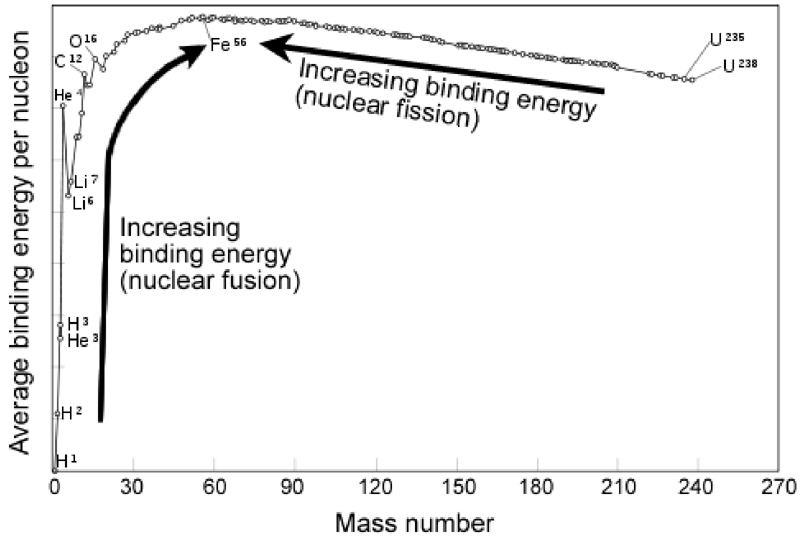
In the fission of uranium-235, a slow-moving neutron collides with an unstable U-235 nucleus, causing it to split into a krypton-93 nucleus and a barium-140 nucleus. The three extra neutrons are emitted. These neutrons may cause the fission of other U-235 nuclei. (Note: In each symbol, the top number is the mass number of the atom and the bottom number is the atomic number, or number of protons.)



Nuclear fission takes place in atomic bombs and nuclear power plants. While fission provides a reliable source of energy that is generally free from air pollution, it does result in large amounts of radioactive waste products that are difficult to safely dispose of.

A nuclear reaction releases energy if it results in a nucleus or nuclei with a greater mass defect per nucleon (and greater binding energy) than the original nucleus. As the graph at right shows, most nuclear fusion reactions release energy for light elements up to Fe-56. Nuclear fission reactions release energy for heavy elements down to Fe-56.

Fusion of elements heavier than Fe-56 and fission of elements smaller than Fe-56 tend to absorb energy.



1. Based on the binding energy graph, which of the following nuclear reactions releases the most energy? (Circle your answer and explain your reasoning below.)
   A. Fusion of hydrogen to produce helium-4
   B. Fission of U-235 to produce Kr-93 and Ba-140
   C. Fusion of C-12 to produce Cr-24
   D. Fission of Fe-56 to produce two Si-28 atoms

   Explanation: ____

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2. How are nuclear fission and fusion similar? ______________________________

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   How are these processes different? ______________________________

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3. Based on the binding energy graph, does nuclear fission or nuclear fusion have the potential to release more energy? Explain your answer.

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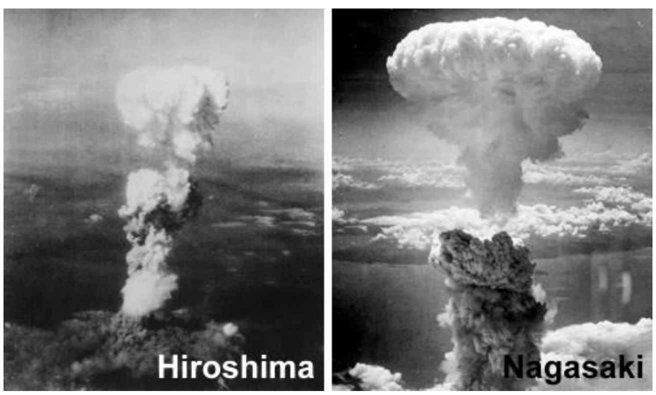
4. Examine the fusion and fission reactions described in the **Nuclear fusion** and **Nuclear fission** sections. Are any particles destroyed and turned into energy in these examples? Explain. (Hint: Count the particles in the reactants and products of each nuclear reaction.)

Activity E: Applications of mass-energy equivalence

Shortly after Einstein's discovery of mass-energy equivalence, the atomic nucleus was discovered by Ernest Rutherford, Hans Geiger, and Ernest Marsden. In the decades that followed, scientists gained an understanding of nuclear structure and processes and began to realize the tremendous energies that were associated with the nucleus. The first nuclear fission experiments were carried out by Enrico Fermi in 1934, but the results of these experiments were not understood until Otto Hahn and Lise Meitner analyzed a similar experiment in 1938.



Over 200,000 civilians were killed by the atomic bombs that destroyed Hiroshima and Nagasaki.

As the world plunged into the maelstrom of World War II, efforts were begun in both Germany and the United States to produce a nuclear bomb based on fission. The effort eventually fizzled in Germany, but the US-based Manhattan Project succeeded in developing the first atomic bombs.

In August of 1945 atomic bombs were dropped on the two Japanese cities: Hiroshima and Nagasaki. The bombs devastated these cities and hastened the Japanese surrender on August 15, 1945.

The Cold War between the United States and the Soviet Union was marked by a nuclear arms race in which each country developed larger and more destructive nuclear weapons, including the hydrogen bomb in 1953. A hydrogen bomb uses a fission reaction to trigger the fusion of hydrogen, resulting in bombs that up to 3,000 times as powerful as the "Little Boy" bomb that was dropped on Hiroshima.

The 1950s also saw the development of a more peaceful use for fission reactions: nuclear power. A nuclear power plant uses controlled fission reactions to boil water and power turbines that produce electricity. Despite the danger of producing energy this way, nuclear power has spread across the globe and is a major source of power in many countries.

Nuclear fusion has the potential to produce much more energy than nuclear fission, but it also requires much higher temperatures to initiate the reaction. Stars are powered by nuclear fission in their cores. Humans have initiated uncontrolled nuclear fusion reactions in hydrogen bombs but have not yet been able to produce nuclear fusion in a controlled setting that would allow the production of usable energy.

1. **Summarize:** Explain how the idea that a spaceship's measured mass increases as it goes faster is related to the generation of electricity in a nuclear power plant. (Attach additional pages if necessary.)

2. **Investigate further:** Suppose you were the chief energy advisor for the president of the United States. The president is considering expanding our use of nuclear power, but wants to know more before going ahead with his plan.

   Research the pros and cons of nuclear power from the perspectives of economics, public safety, and environmentalism. Write a brief report for the president that summarizes these issues, and then give your recommendation based on your analysis of these issues.