Name: ______________________________ Date: ______________________

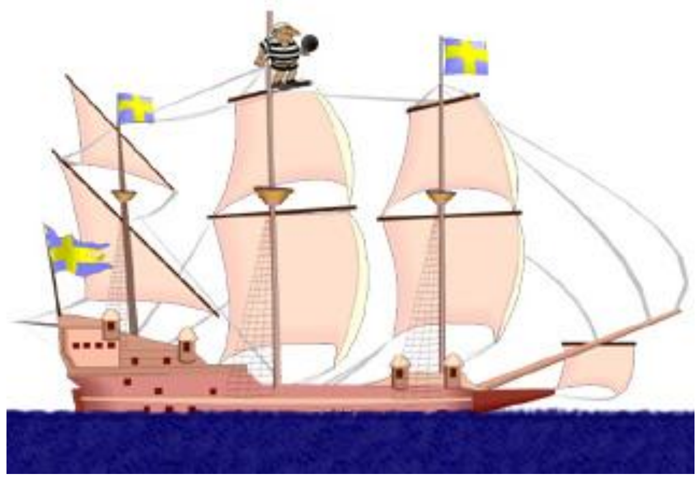
# Guided Learning: Special Relativity

**Learning goals**
After completing this activity, you will be able to …

- Identify the frame(s) of reference that apply to a given situation.
- Explain why the invariance of the speed of light requires a new theory of time and space.
- Describe the consequences of special relativity for time and length measurements.
- Derive and the formula for time dilation. (Extension)

**Vocabulary:** aether, electromagnetic wave, frame of reference, inertial frame of reference, length contraction, principle of relativity, special relativity, time dilation

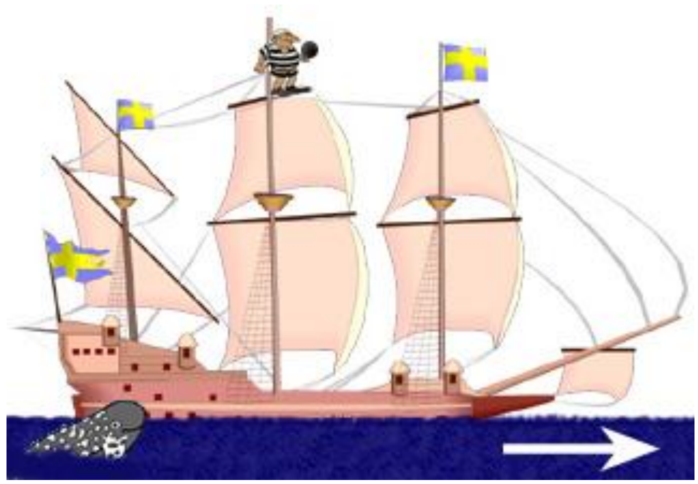
**Warm-up questions**
A sailing ship is traveling at a steady speed through light breezes and relatively calm seas. Sailor Bjorn stands atop the crow's nest, keeping a lookout for pirates, whales, and mermaids. Bored, he decides to do an experiment. He pulls a small cannonball out of his pocket and drops the ball. Where will the ball land?



1. On the top picture at right, draw the path of the ball from the perspective of Bjorn the sailor.

   Describe the path here: ________

   ___________________________



2. A curious whale happened to be watching the experiment as the ship sailed by. On the bottom picture, draw the path of the ball from the perspective of the whale.

   Describe the path here: ________

   ___________________________

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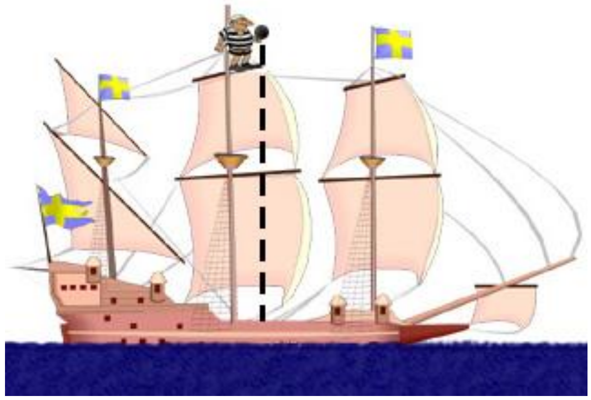
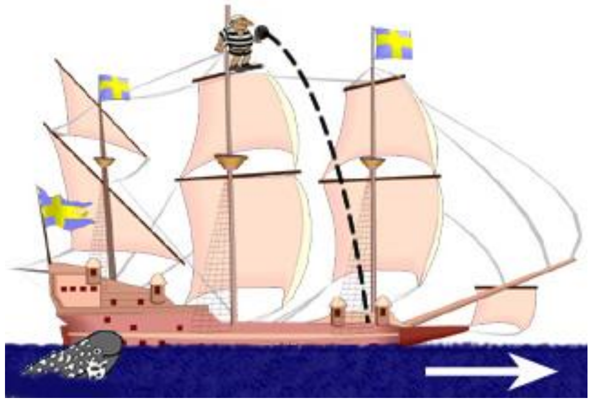
**Combining motions**

If Bjorn's ship was standing perfectly still, the cannonball would fall straight down and land at the base of the mast, as shown at right. But what would happen if the ship were moving to the right?

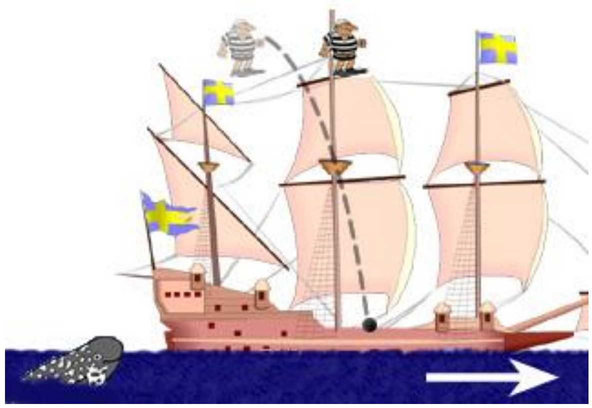


In this case, the cannonball in Bjorn's hand would be moving to the right at the same rate as the ship, Bjorn, and the mast. As the cannonball fell, its trajectory would look like a parabola, as shown in the middle picture at right.



However, as the cannonball fell, because its rightward motion is the same as the rightward motion of the ship and mast, it would still land in the same spot at the base of the mast!

Because the cannonball was on the moving ship, it was moving steadily to the right before it was dropped. After it was dropped, it continued to move at a constant velocity to the right as it was accelerated downward by gravity. Thus the total motion of the ball is simply the sum of its initial motion and its downward free fall.



The observed motion of the ball depends on the **frame of reference** of the observer. A frame of reference is the background that is assumed to be stationary when measuring motion. From Bjorn the sailor's perspective, the ship is standing still and the ball drops straight down to the bottom of the mast. From the whale's perspective, the ship and the ball are moving to the right as the ball falls. While each reference frame is equally valid, the choice of reference frame depends on the observer.

?

If the ship is moving to the right as the cannonball falls, why doesn't the cannonball fall straight down and land behind the mast? If possible, discuss your answer with your classmates and teacher.

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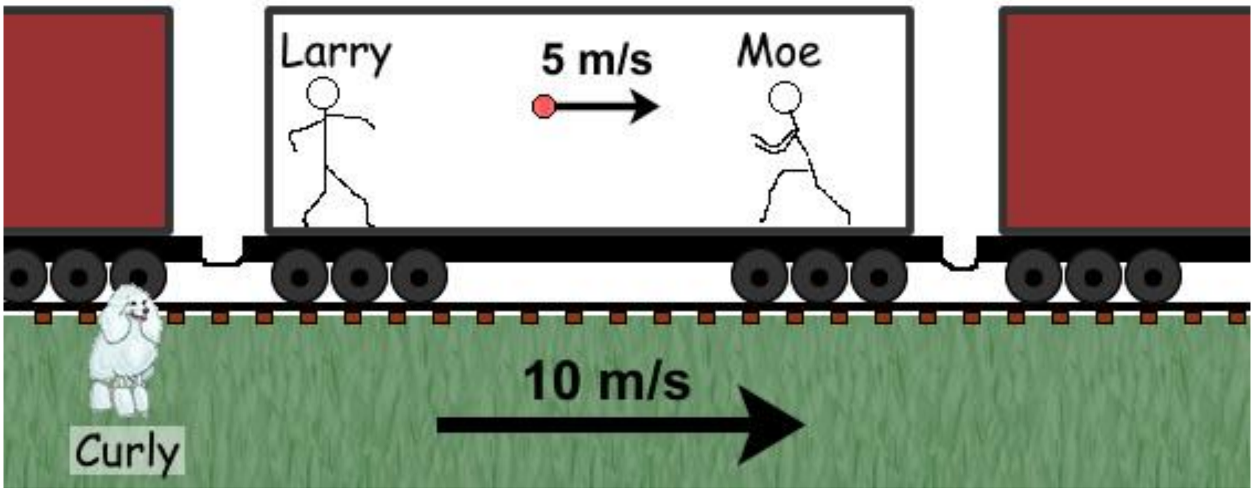
**Playing catch on a train**

The concept of frames of reference was first described by Galileo Galilei in 1632. Galileo also came up with the idea that multiple motions can be simply added together. The idea that physical laws are true in any **inertial frame of reference** is called the **principle of relativity**. (Note: Inertial frames of reference are in constant motion relative to one another.)

Consider the case of two kids, Larry and Moe, riding a train and playing catch in an empty car. The train is moving to the right at 10 m/s. The kids can throw the ball at a speed of 5 m/s. The side of the car is transparent and a poodle named Curly can see the boys playing inside.




1. What is the speed and direction of Larry's throw to Moe, according to Moe? (Hint: Moe's frame of reference is the train.) ______________________________________

2. What is the speed and direction of Moe's throw to Larry, according to Larry?

   ____________________________________________________________

3. What is the speed and direction of Larry's throw to Moe, according to Curly? Explain.

   ____________________________________________________________

   ____________________________________________________________

4. What is the speed and direction of Moe's throw to Larry, according to Curly? Explain.

   ____________________________________________________________

   ____________________________________________________________

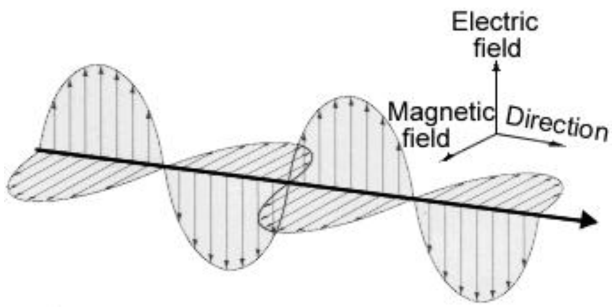
## The speed of light

By the late 1800s, physicists thought they had a good understanding of the world. Newton's laws of motion and universal gravitation seemed to be a complete description of mechanics, even allowing astronomers to deduce the existence of new planets in the solar system. Huge strides had been made in electricity and magnetism, culminating with the publishing of Maxwell's equations in 1862. By the end of the century, many physicists wondered if there were any great ideas left to be discovered.

Boy, were they in for a surprise!

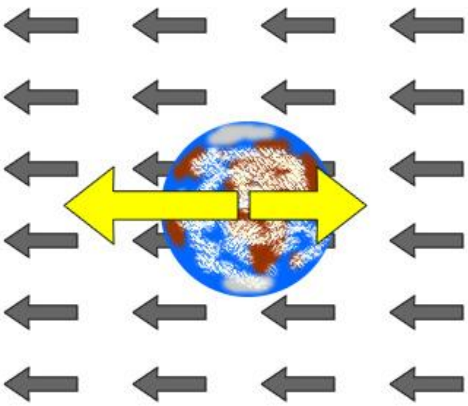
The first indication that something was wrong came from Maxwell's equations, which provided a complete description of electromagnetic force. The third equation, Faraday's law, describes how a changing magnetic field induces an electrical field.

The fourth equation, Ampere's law, describes how a changing electrical field induces a magnetic field. Maxwell realized that these equations implied that it was possible to create a self-sustaining **electromagnetic wave**, which Maxwell realized was light. An electromagnetic wave, which consists of perpendicular fluctuating electric and magnetic fields, is shown at right.




Not only did Maxwell's equations unify electromagnetism and optics by showing that light was an electromagnetic wave, but they yielded a prediction for the exact speed of light, *c*. that was *independent of the motion of the light source*.

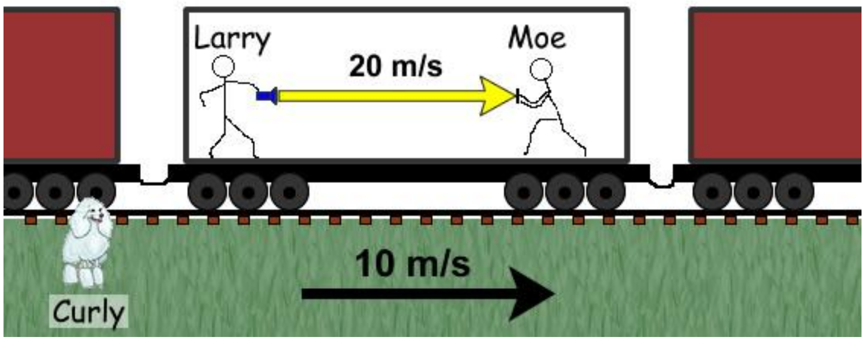
At first, the constancy of the speed of light was not a problem because physicists at the time assumed that a mysterious substance, called **aether**, filled the universe and supported the propagation of light waves, just as air supports the propagation of sound waves. (Sound waves cannot travel through a vacuum.) If the speed of light was only measured relative to the universal frame of reference of the aether, a constant speed of light was not a problem.



Scientists soon realized they could test the aether theory by measuring the speed of light in different directions. Earth travels at a high speed around the Sun, presumably swimming through a sea of aether as it traveled. Because of Earth's motion, scientists predicted that the measured speed of a beam of light projected in the same direction as Earth moved would be equal to the speed of light minus the speed of the Earth. By the same reasoning, the measured speed of a beam of light in the opposite direction would be equal to the speed of light plus the speed of Earth. This is shown in the diagram at left.

However, despite numerous measurements, scientists detected *no* differences in the speed of light. Gradually, scientists began to realize that the aether did not exist: space is a vacuum. Furthermore, the speed of light in a vacuum *is the same in all inertial reference frames*. This remarkable fact sets the stage for the revolutionary theories that were coming.

To understand why the constancy of the speed of light is a problem for Galilean relativity, let's go back to Larry and Moe on the train. Suppose that Larry is shining a flashlight at Moe instead of throwing a ball. The train's velocity is still 10 m/s. For simplicity, assume the speed of light is 20 m/s. (Its actual speed is 299,792,458 m/s.) As usual, Curly is watching from outside the train.

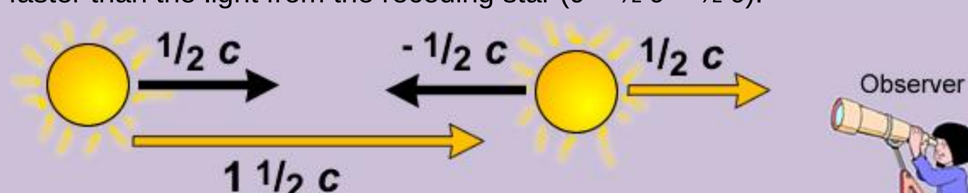



1. What is the speed and direction of Larry's flashlight beam, according to Moe? (Hint: Moe's frame of reference is the train.) __________________________________________

2. According to Galilean relativity (where velocities add up), what is the speed and direction of Larry's flashlight beam, according to Curly? ____________________________________

**What if the speed of light in a vacuum was *not* constant?**

Imagine you are an observer watching two stars on a collision course. One star is moving toward you at half the speed of light ($v = ½\ c$) and the other star is moving away from you at half the speed of light ($v = –½\ c$). If light worked like other objects (e.g. baseballs) you would expect light from the approaching star to have a velocity of $c + ½\ c$, or $1½\ c$. This would be three times faster than the light from the receding star ($c – ½\ c = ½\ c$).

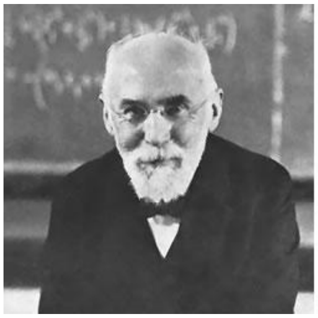


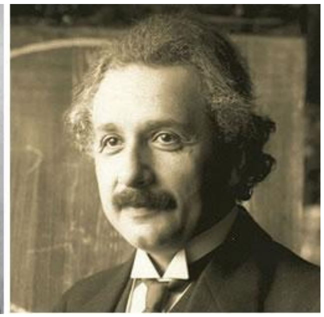

Now imagine the stars approaching one another. As the stars drew close, you would see light from the approaching star long before you saw light from the receding star. Focusing on the spot of the collision itself, you would only see the approaching star in your field of view before the sudden explosion. It would look like the approaching star blew up by itself!

This contradiction is resolved by the fact that the speed of light *is* constant in all reference frames. The light from the approaching star has a velocity of *c*, as well as the light from the receding star. The observer sees the two stars approach each other and collide, which is just what the stars actually do.

**Special relativity**
As you have found, according to Galilean relativity, Curly and Moe have very different measurements for the speed of Larry's flashlight beam. But this contradicts the idea that comes from Maxwell's equations, that the speed of light is constant in *all* reference frames.

The problem was solved thanks to contributions by three scientists: the Dutch physicist Hendrik Lorentz (1853–1928), the French mathematician Henri Poincaré (1854–1912), and of course the German physicist Albert Einstein (1879–1955). Their solution is known as **special relativity**.





**Hendrik Lorentz** **Henri Poincaré** **Albert Einstein**

The theory of special relativity is based on two postulates:

- All inertial frames of reference are equally valid.
- The measured speed of light in a vacuum is always the same in any reference frame.

The first postulate is unchanged from the principle of relativity originally proposed by Galileo. All laws of physics should apply equally well, no matter what inertial frame of reference you have chosen. (Recall that an inertial frame of reference is one that is moving at a constant speed relative to other reference frames.) For example, Larry and Moe are riding on a train that is going at a constant velocity of 20 m/s compared to Curly, who is sitting on the ground. In Larry and Moe's frame of reference, which is equally valid, Curly and the ground are moving at 20 m/s to the left while the train is standing still.

A major consequence of the second postulate, the constancy of the speed of light, is that time itself is not universal—like speed and other measurements, time depends on the frame of reference of the observer. This effect is called **time dilation**. In the example of Larry, Moe, and Curly, suppose the train car has a width of 20 meters. Recall that the speed of the train is 10 m/s and the speed of light is 20 m/s.

On the train, Larry and Moe agree that Larry will turn on the flashlight at exactly 12:00:00 noon. Moe sees the light at exactly 12:00:01 noon. Moe concludes that the light went a distance of 20 meters in one second and calculates a speed of 20 m/s.

Outside the train, Curly the poodle sees the light leave Larry's flashlight and hit Moe. At the same time, the train is moving in the same direction as the beam of light, so Curly sees the light travel farther than 20 meters before it hits Moe. In order to get the same value for the speed of light, Curly must get a longer measurement for the *time* it takes the light to reach Moe. In other words, in Curly's point of view, Curly's clock runs faster than Moe's clock! If Curly were to observe Larry and Moe, their actions would appear to be in slow motion. Strangely, the reverse is also true: from Larry and Moe's frame of reference, Curly's clock appears to run slow and Curly appears to move in slow motion.

Besides time dilation, the constancy of the speed of light has many other odd effects. For example, as the train approaches the speed of light, its length (observed from outside the train) becomes shorter, an effect called **length contraction**. Of course, from the perspective of the people in the train, the *outside* world becomes compressed. This effect is required by the first postulate—from the perspective of the people on the train, the train is standing still while the outside world travels by at a blinding speed.

When Einstein analyzed the effects of an object emitting light, he was led to a remarkable conjecture: *mass and energy are equivalent*. This idea, summarized in the famous equation $E = mc^2$, will be discussed in more detail in the next Guided Learning activity.

Although the consequences of special relativity may seem bizarre, they have been tested and found to be true by many experiments. For example, in 1971, NASA flew two atomic clocks around the world in opposite directions, using commercial airline flights. As relativity would predict, one of the clocks gained time while the other clock lost time compared to the clock at the U.S. Naval Observatory, thus demonstrating the effects of motion on time.

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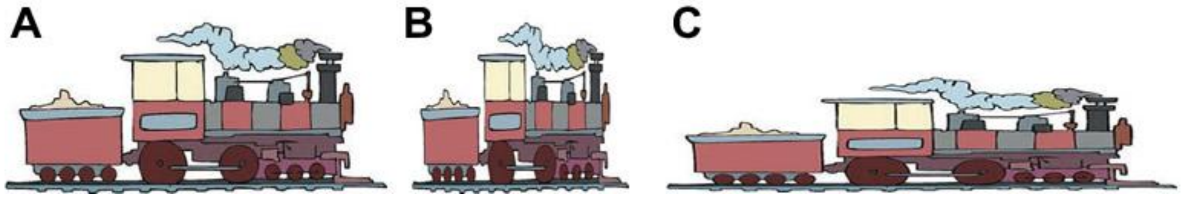
1. A stationary train looks like picture **A** below. Which picture shows what the same train would look like if it sped past you at nearly the speed of light?




Explain: ______________________________________________________________

______________________________________________________________________

2. Why does the constancy of the speed of light imply that the measurement of time changes in different inertial reference frames? Explain in your own words.

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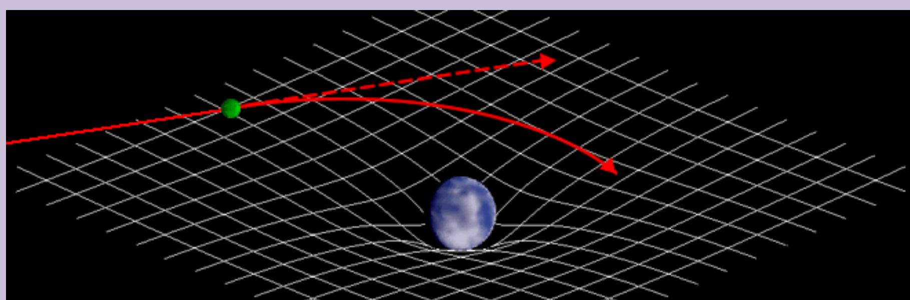
**Appendix: A brief note on general relativity**

The theory of special relativity only deals with *inertial* reference frames, that is, reference frames that are moving at constant velocities. Einstein sought to broaden special relativity to include constant acceleration and gravity. This project sprang from a famous thought experiment in which Einstein realized that a person in a windowless box who felt a downward force would have no way of telling whether he was feeling the gravity of a planet or was aboard an accelerating spaceship. Einstein realized that gravity was equivalent to acceleration, and sought a geometrical model to explain gravity.

According to Einstein's general relativity theory, published in 1915, a massive body has the effect of bending the space around it, as illustrated below. This curvature deflects the paths of nearby objects, causing them to orbit the massive object.



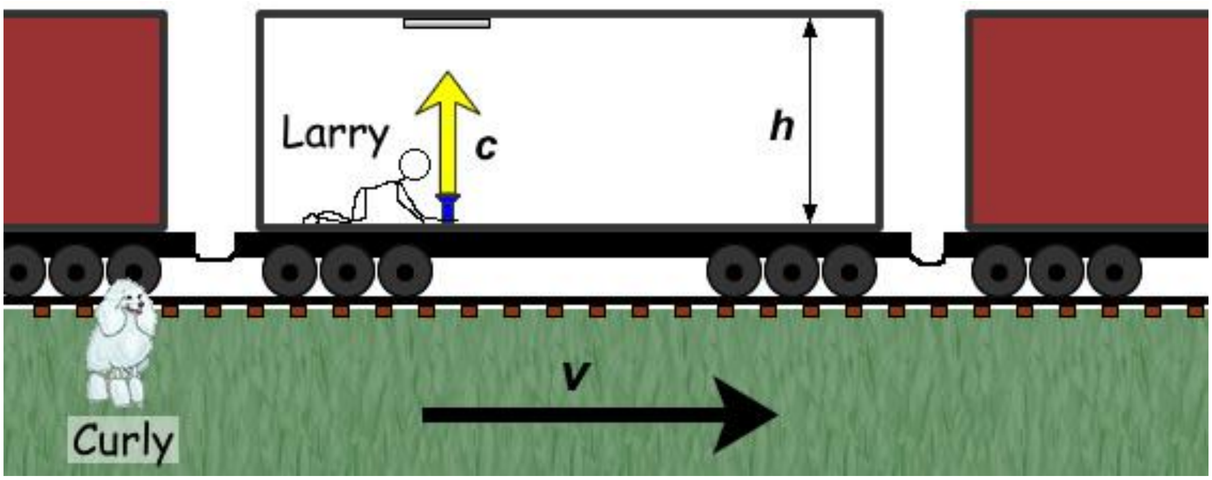
The first solid test of general relativity occurred in during a solar eclipse in 1919. The eclipse allowed scientists to observe stars located behind the Sun. According to general relativity, the gravitational space-time warp caused by the Sun would bend the light coming from these stars, slightly changing their apparent positions. Careful measurements revealed that the starlight was bent precisely as general relativity predicted. These results helped to confirm general relativity and made Einstein and international icon.

Name: ______________________________________ Date: ________________________

# Algebra Extension: Deriving the Time Dilation Equation

**Learning goals**
After completing this activity, you will be able to …
- Derive the time dilation formula.
- Calculate the time dilation that would be observed in various situations.

**Introduction**
In the *Guided Learning: Special Relativity* activity you learned that time dilates on a moving object, but by how much? To solve this problem, let's go back to our friends Larry, Moe, and Curly. Imagine that Larry no longer points his flashlight at Moe, but at a detector on the ceiling instead. Variables are used because the goal is a general solution: *h* is the height of the ceiling, *v* is the speed of the train, and *c* is the speed of light.

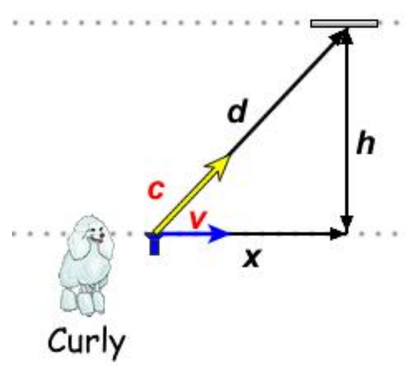



In Larry's frame of reference, we can ignore the speed of the train. The pulse of light from the flashlight must travel a distance of *h* before it hits the ceiling (ignore the length of the flashlight). Assume the beam travels at speed *c* for time *t* to go a distance of *h*.

1. Express *h* in terms of time *t* and speed *c*: *h* = ______

2. From Curly's perspective, the beam travels along a diagonal because the train is moving to the right at the same time the beam is traveling up to the ceiling. Suppose the length of this path is *d*. Assume the beam takes time *t'* to travel a distance *d* at a speed *c*.

   Express *d* in terms of time *t'* and speed *c*: *d* = ______



3. The train is also moving at velocity $v$ and travels a distance $x$ in time $t'$.

   Express $x$ in terms of $v$ and $t'$: $x$ = ______

4. By the theorem of Pythagorus, $d^2 = x^2 + h^2$. Substitute the expressions you derived in questions 1, 2, and 3 above for $d$, $x$, and $h$ in this equation:

5. You should now have the following relationship: $(ct')^2 = (vt')^2 + (ct)^2$

   Solve this equation for $t'$, the time in the stationary frame.

   Show your work:

**Time dilation equation**

If you did the algebra correctly, you should have an expression equivalent to the following:

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

Check with your teacher to see if you are on the right track. (There may be a few algebraic tricks you need to perform to get your equation into exactly this format.) The symbol $\gamma$ (gamma) is often used for the term in the denominator because it appears often in relativistic equations.

1. Suppose the train was going at half the speed of light: $v$ = ½ $c$. On the train, Larry tosses a ball and catches it. This took one second according to Moe, who is also on the train.

   How long did the toss take according to Curly, who is sitting outside the train? ___________

   Show your work:

2. Suppose the train was going at 99% of the speed of light: $v$ = 0.99$c$. Larry tosses the ball to Moe again, and Moe measures a time of 1 second.

   How long did the Larry's toss take according to Curly in this situation? ___________

   Show your work:

3. The fastest spacecraft ever launched was the Helios probe, which traveled approximately 70,000 m/s as it passed close to the Sun. The speed of light is 299,792,458 m/s. Suppose the Helios probe passed by a stationary observer on Earth. The observer can see an accurate clock ticking on the side of the Helios Probe.

   How many seconds on Earth are equivalent to one second on the Helios probe?

   _______________________________________________

   Show your work: