Name: _____ Date: _____

Student Exploration: Theoretical and Experimental Probability

Vocabulary: experimental probability, law of large numbers, outcome, probability, sample space, theoretical probability, trial

Prior Knowledge Question (Do this BEFORE using the Gizmo.)
Carlos, Margaret, and James are playing a board game with a spinner. The spinner goes from 1 to 6. They start by spinning to see who will go first. (Highest number will go first.)

1. First, Carlos spins a 3. How likely do you think it is that Margaret will get a higher number?

   _____

2. Next, Margaret spins a 5. How likely do you think it is that James will get a higher number?

   _____

Gizmo Warm-up
The **probability** of an event is the likelihood that the event will happen. Probability is given as a number that ranges from 0 (impossible) to 1 (certain). You can explore probability using numbered spinners with the *Theoretical and Experimental Probability* Gizmo.

1. To begin, check that the **Number of spinners** is 1, ***Sections*** is 6, ***Number*** is 2, and the = sign is chosen. In this game, a win (a favorable **outcome**) occurs if the spinner lands on 2.

   How likely do you think it is that a player will win the game? Explain. _____

   _____

2. On the **EXPERIMENTAL** tab, click **Run 1 trial**. What was the outcome? _____

3. Click **Clear**. Then, click **Run 10 trials**. How many trials were favorable? _____

4. Click **Run 10 trials** 5 more times so there are a total of 60 trials. How many favorable outcomes did you get out of 60 trials? _____

Activity A: **One-spinner games**	Get the Gizmo ready: • With **Number of spinners** set to **1**, set ***Sections*** to 3, ***Number*** to 1, and the sign to =.	= ≠ > ≥ = 1

1. The set of all possible outcomes make up the **sample space** of an experiment.

   A. What are the possible outcomes of each spin of this spinner? _____

   B. Is each outcome equally likely? _____ How do you know? _____

   _____

   C. Of these outcomes, how many are favorable? _____

   D. What do you think are the chances of a favorable outcome on one spin? _____

   E. How many favorable outcomes do you expect in 100 spins? _____

   F. Click **Run 10 trials** 10 times. How many favorable outcomes occurred? _____

   Was this close to what you predicted? _____

2. Select the **THEORETICAL** tab. The tab shows a table of outcomes. The red numbers show the possible numbers on the spinner, and the blue number represents the selected number. In the table, **Y** represents a favorable outcome, while **N** represents an unfavorable outcome.

   A. How many outcomes are favorable? _____

   B. How many outcomes are listed? _____

   C. What fraction of the total outcomes is favorable? _____

   D. What fraction of the total outcomes is unfavorable? _____

3. Turn on **Show theoretical probabilities**. The table shows the number of favorable and unfavorable outcomes, and the **theoretical probabilities** as fractions and percentages.

   A. What is the theoretical probability of a favorable outcome? _____

   B. What is the theoretical probability of an unfavorable outcome? _____

   C. How do these fractions compare to what you calculated in questions 2C and 2D?

   _____

   D. What is the sum of the favorable and unfavorable probabilities? _____

(Activity A continued on next page)

Activity A (continued from previous page)

4. Turn off **Show theoretical probabilities**. Change the ***Sections*** in the spinner to 7, the ***Number*** to 4, and the sign to ≥.

   A. In this game, what are the favorable outcomes? _____

   B. How many possible outcomes are there? _____

   C. What is the theoretical probability of a favorable outcome? Give your answer as a fraction and as a percentage. _____

      Turn on **Show theoretical probabilities** to check.

5. The **experimental probability** of an outcome is the fraction (or percentage) of times the outcome occurs in an experiment. On the **EXPERIMENTAL** tab, run 100 trials.

   A. How many favorable outcomes occurred? _____

   B. What is experimental probability of a favorable outcome? _____

   C. How did the experimental probability compare to the theoretical probability you calculated above? _____

6. On the **THEORETICAL** tab, turn off **Show theoretical probabilities**. Change the ***Sections*** in the spinner to 10, the ***Number*** to 3, and the sign to <.

   A. As a percentage, what is the probability of a favorable outcome? _____

      Turn on **Show theoretical probabilities** to check your answer.

   B. Run 100 trials. How many outcomes were favorable? _____

   C. How did the experimental probability compare to the theoretical probability? _____

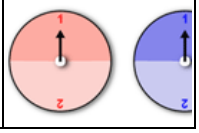
      _____

7. In general, how do you find the theoretical and experimental probabilities of a favorable outcome if there are *n* equally likely outcomes and *p* of them are favorable?

   _____

   _____

   _____

Activity B: **Two-spinner games**	Get the Gizmo ready: • Select the **EXPERIMENTAL** tab. • Next to **Number of spinners**, select 2. Set the ***Sections*** of each spinner to 2, and the sign to [=].	

1. In this game, two spinners are spun. If the spinners show the same number, the outcome is favorable. If the spinners show different numbers, the outcome is unfavorable.

   A. What are the possible outcomes of spinning these two spinners? List outcomes as pairs. For example, (1, 1) is a "1" on each spinner. _____

   B. How many of the outcomes are favorable? _____

   C. What do you think is the theoretical probability of a favorable outcome? _____

   D. On the **THEORETICAL** tab, turn on **Show theoretical probabilities**. Were you correct? _____

2. Turn off **Show theoretical probabilities**. Set up this two-spinner game as described below.
   - Set the red spinner ***Sections*** to 6.
   - Set the blue spinner ***Sections*** to 5.
   - Select [<]. (Favorable outcome is red < blue.)

   In the table to the right, fill in the possible red spinner values down the left, and the possible blue spinner values across the top. For each outcome (pair of numbers), fill in **Y** (favorable) or **N** (unfavorable).

	Possible blue outcomes									
Possible red outcomes										

   A. How many possible outcomes are there? _____

   B. How does the number of possible outcomes relate to the number of sections on each spinner? _____

   C. How many of the outcomes are favorable? _____

   D. What is the theoretical probability of a favorable outcome? _____ Check your answer by turning on **Show theoretical probabilities**.

3. In general, how do you find the theoretical probability of a favorable outcome in a two-spinner game? _____

_____

Activity C: **The law of large numbers**	Get the Gizmo ready: • On the **THEORETICAL** tab, turn off **Show theoretical probabilities**. • Set the **Number of spinners** to 1, ***Sections*** to 10, ***Number*** to 7, and the sign to ≥.	

1. Check that the spinner has 10 sections, the ***Number*** is 7, and the sign is ≥.

   What is the theoretical probability of a favorable outcome? _____

   Turn on **Show theoretical probabilities** to check.

2. On the **EXPERIMENTAL** tab, click **Run 10 trials**. Record the number and percentage of favorable outcomes in the first column of the table below. Click **Clear**. Repeat the experiment seven more times (clicking **Clear** after each one) to complete the table.

Experiment	1	2	3	4	5	6	7	8
Number favorable								
Percentage favorable								

3. Click **Clear**. Now, do the same experiment, but with 100 trials in each experiment. (To run 100 trials, click **Run 10 trials** 10 times.) Be sure to click **Clear** after each experiment. Fill in the table below.

Experiment	1	2	3	4	5	6	7	8
Number favorable								
Percentage favorable								

4. Compare your results in the two data tables above.

   A. Which experiment gave more consistent favorable percentages, 10-spins-per-trial or 100-spins-per-trial? _____

   B. Which experiment gave experimental probabilities that were closer to the theoretical probability? _____

   C. In general, how does the number of trials seem to affect experimental probability?

   _____

   This is an example of the **law of large numbers**. In general, the greater the number of trials, the closer the experimental probability will be to the theoretical probability.