Name: _____   Date: _____

Student Exploration: Triangle Inequalities

Vocabulary: equilateral, inequality, isosceles

Prior Knowledge Questions (Do these BEFORE using the Gizmo.)
The map to the right shows Kara's house, Bob's house, and their school. Kara lives 5 miles from the school and Bob lives 3 miles from the school. (The map is not drawn to scale.)
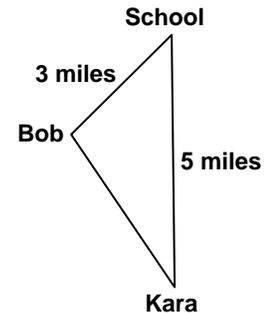
1. Jason guesses that Kara and Bob live 1 mile from each other.

   Is this possible? _____

2. Explain why or why not. _____

   _____

Gizmo Warm-up
In the *Triangle Inequalities* Gizmo, you will explore how the measures of the sides and angles of a triangle are related. You will use the Gizmo to discover important **inequalities** that apply to triangles. An inequality is a relationship in which one quantity is greater than or less than another quantity.

To begin, explore how the vertices and sides of a triangle are labeled. Be sure **Show side lengths** and **Show labels** are turned on.
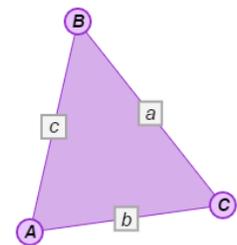
1. Fill in the blanks below with the lowercase label that corresponds to each side.

   $\overline{AB}$ _____ $\overline{BC}$ _____ $\overline{CA}$ _____

2. How are these lowercase labels related to the vertices of the triangle? _____

   _____

3. Vary the triangle by dragging its vertices around. How do you think $a + b$ compares to c?

   _____

Activity A: **Side inequalities**	<u>Get the Gizmo ready</u>: • Be sure **Show side lengths** and **Show labels** are selected.	
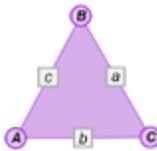
1. In the Gizmo, drag the vertices to make a triangle with sides that are all about equal lengths.

   A. Imagine that your triangle is a map. Do you think *c* or *a* + *b* is the shortest route from

   point *A* to *B*? _____ Select **Compare side lengths** and *c* and *a* + *b* to check.

   B. Write an inequality to describe how *c* is related to *a* + *b*. _____

   C. Watch the values under **Compare side lengths** as you create a variety of triangles.

   Is the inequality you wrote above true for all the triangles you created? _____

2. In the Gizmo, create a situation in which *c* is equal to *a* + *b*.

   A. What do you notice? _____

   B. Is the figure you created still a triangle? _____

3. In the Gizmo, be sure **Compare side lengths** is still selected.

   A. Select *a* and *b* + c under **Compare side lengths**. Create a variety of triangles. Write

   an inequality to describe the relationship between *a* and *b* + *c*. _____

   B. Select *b* and *a* + c under **Compare side lengths**. Create a variety of triangles. Write

   an inequality to describe the relationship between *b* and *a* + *c*. _____

   C. How does the sum of two side lengths of a triangle compare to the third side length?

   _____

   This relationship is known as the *Triangle Inequality Theorem*.

4. Determine if each of the following can be side lengths of a triangle. If not, explain why not.

   A. 2, 6, 11 _____

   B. 8, 8, 15 _____

   C. 13, 16, 29 _____

	Get the Gizmo ready:
Activity B: **Angle inequalities**	- Turn off **Compare side lengths**. - Turn on **Show side lengths** and **Show values**. - Select **Show angle measures**.

1. In the Gizmo, drag the vertices to create a triangle with three different angle measures.

   A. List the angles in order from smallest to largest. _____

   B. Look at the sides opposite each angle. Write the names of the sides in order from

   shortest to longest. _____

   C. Compare the order of the sides and the angles. What do you notice? _____

   _____

   Create a variety of triangles to check that this is always true.

2. An **isosceles** triangle has at least two congruent sides. What do you think is true about the

   angles of an isosceles triangle? _____

   Why? _____

   Check your answer in the Gizmo.

3. An **equilateral** triangle has three congruent sides. What do you think is true about the

   angles of an equilateral triangle? _____

   Why? _____

   Check your answer in the Gizmo.

4. Summarize what you have learned in this activity by completing the statements below.

   A. In any triangle, the longest side is opposite the _____ angle and the

   shortest side is opposite the _____ angle.

   B. In any triangle, the largest angle is opposite the _____ side and the

   smallest angle is opposite the _____ side.

Activity C: **Using triangle inequalities**	Get the Gizmo ready: • Turn off **Compare side lengths**. • Turn on **Show side lengths** and **Show values**. • Select **Show angle measures**.	
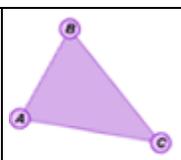
Recall that the following inequalities are true for all triangles:

- The sum of two side lengths is greater than the third side length. (This is the Triangle Inequality Theorem.)

- The longest side is opposite the largest angle and the shortest side is opposite the smallest angle.

- The largest angle is opposite the longest side and the smallest angle is opposite the shortest side.

Use these relationships to solve the problems below.

1. In $\triangle ABC$, $m\angle A = 53°$, $m\angle B = 69°$, and $m\angle C = 58°$. Name the longest and shortest sides.

    Longest side: _____   Shortest side: _____   Check your answers in the Gizmo.

2. The lengths of two sides of a triangle are 3 feet and 9 feet. Find the range for the length of the third side. Explain your reasoning.

3. Farmer John has 23 meters of fencing to build a triangular pig pen. He wants two of the sides of the pen to be 9 meters and 12 meters long. Determine if he has enough fencing to build the pen. Explain your reasoning.

4. Give two reasons why it's impossible to draw $\triangle PQR$ with $m\angle Q = 103°$, $m\angle R = 47°$, $p = 44$, $q = 12$, and $r = 31$.

    Reason 1: _____

    Reason 2: _____